

$(a, b)$  in a rectangular coordinate system, the analytic functions correspond to transformations analogous to conformal transformations, the "angle" between two directions of slopes  $m_1, m_2$  being defined as  $\tanh^{-1}(m_2 - m_1)/(1 - m_1 m_2)$ . Lorentz transformations are a particular case. By adjoining suitably two intersecting lines at infinity, the plane of Clifford numbers may be mapped on a ring-shaped second order surface in projective 3-space just as complex numbers are mapped stereographically on a sphere. The linear transformations of a Clifford variable give geometrical transformations similar to the linear transformations of a complex variable, the metric  $dx^2 - dy^2$  replacing the euclidean. The exponential function  $e^{x+y\Lambda} = e^x(\cosh y + \Lambda \sinh y)$  is not periodic, whereas  $\sin(x+y\Lambda) = \sin x \cos y + \Lambda \cos x \sin y$  and the other trigonometric functions are doubly periodic. Cauchy's integral theorem and other complex variable theorems hold unchanged, or with slight modifications. (Received April 2, 1941.)

323. Nelson Robinson: *A transformation between osculating curves of a rational normal curve in an odd dimensional space.*

The equations of a rational normal curve  $\Gamma_n$  in a linear space of  $n$  dimensions are reduced to a canonical form. Hyperquadrics  $Q$ , containing  $\Gamma_n$ , and osculating curves  $\Gamma_{n-j}$  of  $\Gamma_n$  at a point  $P$  are defined geometrically and their equations derived. By use of a hyperquadric  $Q$ , a transformation involving a generalized null system is set up between osculating curves. This correspondence is proved to exist if and only if the ambient space is of odd dimensions. (Received May 14, 1941.)

#### STATISTICS AND PROBABILITY

324. Alfred Basch: *A contribution to the theory of multiple correlation.*

If  $r_{xy}$  and  $r_{xz}$  are the correlation coefficients between  $x$  and  $y$ , and  $x$  and  $z$ , then  $r_{yz}$  lies between the limits  $r_{xy}r_{xz} \pm k_{xy}k_{xz}$ , where  $k_{xy} = (1 - r_{xy}^2)^{1/2}$ ,  $k_{xz} = (1 - r_{xz}^2)^{1/2}$  are the alienation coefficients. In a geometrical representation where  $r_{xy}$  and  $r_{xz}$  are the rectangular coordinates, the ellipses inscribed in the square with the sides  $r_{xy} = \pm 1$ ,  $r_{xz} = \pm 1$  are the loci of constant lower and constant upper limits of  $r_{yz}$ . Rays parallel to the three coordinate axes give the three contour ellipses of the standard ellipsoid. From these three contour ellipses can be obtained the three correlation coefficients. If two of the contour ellipses are given, then the third must satisfy restricting conditions in agreement with the limits given for the third correlation coefficient. The intersection ellipses of the standard ellipsoid with the coordinate planes are characteristic for the coefficients of the correlation between two variables, freed from the influence of the third. The "limit cases" correspond to the degeneration of the standard ellipsoid,  $r_{yz}, x = r_{xy}, z = r_{xz}, y$ . In these limit cases there exists a functional relation between the three variables. The "middle case,"  $r_{yz} = r_{xy}r_{xz}$  is equivalent to  $r_{yz}, x = 0$ . The standard ellipsoid intersects in this case the  $yz$ -plane in an ellipse, whose chief axes are the coordinate axes. (Received April 3, 1941.)

#### TOPOLOGY

325. H. A. Arnold: *Homology in set-intersections, with an application to  $r$ -regular convergence.*

$A$  and  $B$  are closed subsets of compact space  $R$ . Using Vietoris cycles the following lemmas are proved: (1) If the complete  $r$ -cycle  $\gamma^r$ , carried by  $A$  is  $\sim 0$  in  $A+B$ , then