another line L. Any such set may be considered to be analogous to the velocity systems as developed by Kasner. Any line transformation, not preserving all parallel pencils of lines, converts exactly one dual-velocity system into a dual-velocity system. The group preserving all dual-velocity systems is $X=\phi(x)$, $Y=y\psi(x)+\chi(x)$. This is the contact group leaving invariant the set of all dual-isothermal families. Examples of dual-velocity systems are equitangential, dual-natural, Δ, and dual-Γ families. Characterizations of these are obtained by the correspondence between the lines l and L mentioned above. Any dual-velocity system contains exactly ∞^2, ∞^1, one, or zero dual-isothermal families. Finally, the invariant theory of dual-velocity systems under both the dual-isothermal and equilong groups is developed. A dual-analogue of natural family has been discussed in an earlier paper. (Received February 11, 1941.)

266. L. J. Savage: *Distance spaces.*

By a distance space is meant a set M of elements $p, q, \cdots$ over which is defined a real valued function $D(p, q)$ such that $D(p, q)=D(q, p)$, and $D(p, p)=0$. $D(p, q)$ may be thought of as the square of the distance from p to q. This concept is some generalization of metric space. A vector space V over which a scalar product $x\cdot y$ is defined can be considered as a distance space by setting $D(x, y)=(x-y)\cdot(x-y)$. For every distance space M there is a "smallest" scalar product space $V(M)$ in which M is imbeddable. An interesting class of distance spaces is that of differentiable manifolds over which a $D(p, q)$ is so defined as to be suitably differentiable when considered as a function of the coordinates. Because of the remark about scalar product spaces these differentiable distance manifolds can be handled much like differentiable submanifolds of euclidean space. In particular the concepts of regularity, tangent-flat, and first and second fundamental form, can be extended to them. Finally there are theorems connecting the possibility of imbedding such manifolds into euclidean and pseudo-euclidean spaces with certain restrictions on the second fundamental form. (Received March 13, 1941.)

267. R. K. Wakerling: *On the rational loci of ∞^1 $(\rho-1)$-spaces in r-space.*

The representation upon a ρ-space of the hypersurface W_ρ^n in S_r, which is the rational locus of ∞^1 $(\rho-1)$-spaces, is investigated in this paper. The hyperplane sections of W_ρ^n are represented in S_ρ by a system of hypersurfaces $V_{\rho-1}^\nu$ passing through a given $(\rho-2)$-space $\nu-1$ times, and having in common σ simple points. Some of the properties of W_ρ^n are discussed together with those of its projection upon a $(\rho+1)$-space. A special case of the transformation between two r-spaces is given, and the paper is concluded with a brief note on rational ruled surfaces of order $r-1$ in S_r. (Received March 8, 1941.)

Statistics and Probability

268. A. H. Copeland: *If.*

If one attempts to apply Boolean algebra to the theory of probability, he discovers that it is inadequate for the treatment of conditional probabilities, selections, and observations—all three of which are of prime importance in the modern theories of probability and statistics. In a number of recent formalizations of the theory of probability an additional operator (or logical constant) "if" has been introduced in order to handle conditional probabilities. Selections and observations were not introduced