ON THE ANALOGUE FOR DIFFERENTIAL EQUATIONS OF THE HILBERT-NETTO THEOREM

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If

$$F_1, \cdots, F_r \tag{1}$$

is a finite system of differential polynomials in the unknowns $y_1, \cdots, y_n$, and if G is a differential polynomial which is annulled by every solution of the system (1), some power G^p of G is a linear combination of the F_i and their derivatives of various orders, with differential polynomials for coefficients. This analogue of the Hilbert-Netto theorem was proved by J. F. Ritt[1] for forms with meromorphic coefficients, and by H. W. Raudenbush[2] for the case of coefficients belonging to an abstract differential field. In these proofs it is shown that the denial of the existence of the exponent p, above, of G leads to a contradiction; no constructive method for obtaining admissible values of p is given. The object of the present note is to present a new proof of the analogue, for the case of meromorphic coefficients, which is entirely constructive and produces a definite G^p as described above.

Our proof will be based on the considerations in Chapters V and VII of A.D.E. In Chapter VII, the problem of obtaining G is reduced to the problem of determining unity as a linear combination of the F_i in (1) and their derivatives, in the case in which (1) has no solutions. In Chapter V it is shown how to decide, in a finite number of steps, whether or not (1) has solutions. Our problem thus assumes the following form: *Given that* (1) *has no solutions, it is required to express unity as a linear combination of the F_i and their derivatives.*

We assume that (1) has no solutions and proceed to examine the algorithm developed in §§65–67 of A.D.E. Adjoining to (1) a finite number of linear combinations of the F_i and their derivatives, we obtain a system Σ, devoid of solutions, with a basic set

$$A_1, \cdots, A_q \tag{2}$$

which has the property that the remainder of every form in Σ with respect to (2) is zero. If (2) consists of a single form A which is an

[1] Ritt, J. F., *Differential Equations from the Algebraic Standpoint*, chap. 7, referred to below as A.D.E. American Mathematical Society Colloquium Publications, vol. 14, 1932.

[2] Raudenbush, H. W., *Ideal theory and algebraic differential equations*, Transactions of this Society, vol. 36 (1934), pp. 361–368.