# ON DERIVATIVES OF ORTHOGONAL POLYNOMIALS. II[1]

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It has been shown [1, 2, 7][2] that if $\{y_n(x)\}$ is a set of orthogonal polynomials whose derivatives $\{y_n'(x)\}$ also form an orthogonal set, then the original set $\{y_n(x)\}$ are either Jacobi, Laguerre, or Hermite polynomials. In the papers referred to, the ordinary definition of orthogonal polynomials was used, so of course the result was based on this definition. Since then, however, there has appeared [3, 5][3] a definition of generalized orthogonal polynomials[4] and it is our purpose here to find what G.O.P. sets have G.O.P. derivatives. Besides the classical polynomials, we shall find another class of polynomials which have this property. Of course, this new class will be G.O.P., but not ordinary orthogonal polynomials.

DEFINITION. *Given a set of constants* $\{c_n\}$ *subject to the condition*

$$\Delta_{n+1} = \begin{vmatrix} c_0 & c_1 & \cdots & c_n \\ c_1 & c_2 & \cdots & c_{n+1} \\ \cdot & \cdot & \cdots & \cdot \\ c_n & c_{n+1} & \cdots & c_{2n} \end{vmatrix} \neq 0, \qquad n = 0, 1, \cdots,$$

*the polynomials*

$$y_n(x) = \begin{vmatrix} c_0 & c_1 & \cdots & c_n \\ c_1 & c_2 & \cdots & c_{n+1} \\ \cdot & \cdot & \cdots & \cdot \\ c_{n-1} & c_n & \cdots & c_{2n-1} \\ 1 & x & \cdots & x^n \end{vmatrix}$$

*form a set of G.O.P.*

It has been shown [5, 4] that there exists a function of bounded variation $\psi(x)$ (hence infinitely many), such that

[1] Presented to the Society, April 26, 1940, under the title *Orthogonal polynomial solutions of a certain fourth order differential equation.*

[2] The numbers in square brackets refer to the bibliography at the end.

[3] So far as this author knows, the idea of generalized orthogonal polynomials originated with I. M. Sheffer, who suggested its use in [3].

[4] Hereafter we shall use the abbreviation G.O.P. for generalized orthogonal polynomials.