4. **Other cases.** The corresponding theory for a non-algebraic curve is complicated by the fact that the number of polynomials of the nth degree in the orthogonal system increases with n, but is compensatingly simplified by the observation that the representation corresponding to (2) holds for *all* values of x and y, so that (except for the assumption that the domain of orthogonality is all on one side of the line) the points where the straight line meets the curve are no longer a matter of special concern. If (x_ν, y_ν) are any $n+1$ distinct points on the line, independent polynomials of the nth degree orthogonal to every polynomial of lower degree with respect to the composite weight function are given by $K_n(x_\nu, y_\nu, u, v)$.

A similar conclusion holds for orthogonality on a two-dimensional region.

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NOTE ON AN INEQUALITY OF STEINER[1]

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Let Q denote the unit square $0 \leqq x, y \leqq 1$. If $f(x, y)$ be any function defined and continuous on Q, the relation $z=f(x, y)$ yields a continuous surface defined over Q. The Lebesgue area[2] of this surface will be denoted by $L(f)$. Let $z=f_1(x, y)$, $z=f_2(x, y)$ be two continuous surfaces defined over Q; then clearly $z=[f_1(x, y)+f_2(x, y)]/2$ is a continuous surface defined over Q. The *inequality of Steiner*[3] states that $L([f_1+f_2]/2) \leqq [L(f_1)+L(f_2)]/2$. McShane[4] obtained interesting and important results concerning the situation where the sign of equality holds in this relation. In this note we improve his results and, in a sense, give them a final form.

In order to emphasize and to clarify what is significant and interesting in the results of McShane and in our improvements thereon, we remind our reader of a few facts concerning the Lebesgue area.[5] Given a continuous surface $z=f(x, y)$ defined over Q; if $L(f)$ is finite then the partial derivatives f_x and f_y exist almost everywhere in Q, the integral $\iint_Q [1+f_x^2+f_y^2]^{1/2} dxdy$ exists, and the relation

[1] Presented to the Society, April 13, 1940.

[2] See S. Saks, *Theory of the Integral*, Warsaw and Lwów, 1937, chap. 5, for the facts used in this paper concerning the Lebesgue area.

[3] E. J. McShane, *On a certain inequality of Steiner*, Annals of Mathematics, (2), vol. 33 (1932), pp. 125–138.

[4] Loc. cit.[3]

[5] Cf.[2]