74. S. B. Myers: *Complete Riemannian manifolds of positive mean curvature.*

The author has proved previously that if, on a complete n-dimensional Riemannian manifold M, the curvature at every point and with respect to every pair of directions is greater than a fixed positive constant, then M is closed (compact) and so is its universal covering manifold. In the present paper the same conclusions are drawn from the weaker hypothesis that the mean curvature of M at every point and with respect to every direction is greater than a fixed positive constant. In particular, a complete space of constant positive mean curvature is closed, and so is its universal covering manifold. Such spaces are important in the general theory of relativity. (Received November 25, 1940.)

Statistics and Probability

75. G. A. Baker: *Fundamental distributions of errors for agricultural field trials.*

Evidence from various sources is presented which shows that the fundamental error distribution for yield trials is represented by $[1/ab(t_1-t_0)]\int_0^a\int_0^b\int_{t_0}^{t_1}[1/\sigma(x, y, t)]$ $\exp -\frac{1}{2}\{[\xi-f(x, y, t)]^2/\sigma^2(x, y, t)\}dtdydx$ where the integrals may be Stieltjes integrals. Under certain conditions the fundamental error distribution can be expressed as a Gram-Charlier series, but very rarely, if ever, as a normal distribution. For comparison with analysis of variance results based on the normal theory, the distribution of the ratio of independent estimates of the second moments of samples, if the fundamental distributions are Gram-Charlier series, are given. Similar considerations show that the distributions of the numbers attacked in field trials can rarely be represented by Poisson or binomial distributions as is usually assumed. (Received October 22, 1940.)

76. G. A. Baker: *Maximum likelihood estimation of the ratio of the components of nonhomogeneous populations.*

Let $f(x) = [1/(1+k)](f_1(x)+kf_2(x))$, $e \leqq x \leqq f$, $k>0$ and $k<\infty$, where $f_1(x)$ and $f_2(x)$ are probability functions. The problem is to find the maximum likelihood estimate of k, say $\bar{k}$. If $f_1(x)$ and $f_2(x)$ are rectangular with equal ranges that partially overlap, then the probability of a value of $\bar{k}=w/u$ (where u is the number of individuals drawn from the nonoverlapped interval of $f_1(x)$, w is the number of individuals drawn from the nonoverlapped interval of $f_2(x)$ and v is the number of individuals drawn from the interval overlapped by $f_1(x)$ and $f_2(x)$) is $(n!/u!v!w!)(p_1)^u(p_2)^v(p_3)^w$ where the p_i's are the probabilities of coming from the respective intervals. The cases for which $u=n$, $v=n$, $w=n$, $u=0$, $w=0$ are excluded because $\bar{k}$ is then indeterminate. Hence, the probability of a determinate value of $\bar{k}$ is $P=1+(p_2)^n-(p_2+p_3)^n-(p_1+p_2)^n$. The estimates of k are biased. (Received October 22, 1940.)

77. G. F. McEwen: *Statistical problems of the range divided by the mean in samples of size n.*

Certain quantitative climatological studies are based upon the "precipitation ratio" or ratio to the mean annual rainfall of the difference between the maximum and minimum annual rainfall corresponding to the standard number of years. Available observations correspond to various values of the number of years n. Accordingly it is necessary to compute the precipitation ratio I corresponding to a standard number