

Reelle Funktionen, Part I. By C. Carathéodory. Leipzig, Teubner, 1939. 6+182 pages. RM 11.20.

This volume is Part I of a projected three volume treatise on real functions to be distributed under the following titles: Part I, Numbers, Point Sets, Functions; Part II, Theory of Measure and The Integral; Part III, as yet without a title, but presumed to treat of derivatives and applications.

These volumes will replace the author's one volume, *Vorlesungen Über Reelle Funktionen*, first completed in 1917 and later reissued in 1927 with slight changes. It will be recalled that this earlier volume is written in the elegant axiomatic style for which the author is noted. Among its outstanding features are an abstract treatment of measure and a very full treatment of Lebesgue integration.

This volume preserves much of the material in the first 230 pages of the older treatise. A considerable rearrangement of material has been made which adds to the attractiveness of the book. But examples are given only when required to show the necessity for the conditions governing certain theorems. The reviewers believe that the addition of illustrative examples would have been appreciated by many readers.

Some of the essential changes and additions will be taken up by chapters.

Chapter I—Real Numbers. Denumerability and non-denumerability treated in Chapter II of the older treatise here follows an axiomatic introduction to real numbers.

Chapter II—The Limit Notion. A discussion of Cantor's diagonal process is added to the older chapter on limits.

Chapter III—Point Sets in Euclidian Space. The treatment of open and closed sets also involves an iteration process of forming closures and interiors. An application is made to boundary theory.

Chapter IV—Normal Covering Sequences and the Theory of Connectedness. A new feature is the treatment of connectedness of arbitrary sets.

Chapter V—Functions. No essential change over the older volume save for omission of a discussion of functions of bounded variation (deferred to Part II) and the inclusion of a section on continuous monotonic functions in which is introduced a transformation function

$$x = \lambda(y) = \frac{1+y+|y|}{2(1+|y|)}$$