

# CESÀRO SUMMABILITY OF ORDINARY DOUBLE DIRICHLET SERIES

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**1. Introduction.** The purpose of this paper is to obtain some results in the Cesàro summability of ordinary double Dirichlet series similar to those obtained by H. Bohr<sup>1</sup> for the simple series. As is well known a double sequence  $\{S_{mn}\}$  may tend to a finite limit as  $m, n \rightarrow \infty$ <sup>2</sup> without  $S_{mn}$  being a bounded function of  $m$  and  $n$ . In order to avoid difficulties in this respect and to obtain results analogous to those for simple series, the discussion will usually be restricted to bounded sequences.

Let  $\sum_{m,n=1}^{\infty} u_{mn}$  be a double series of constant terms. Set

$$(1) \quad \begin{aligned} S_{mn}^{00} &= \sum_{i=1}^m \sum_{j=1}^n u_{ij}, & S_{mn}^{10} &= \sum_{i=1}^m S_{in}^{00}, & S_{mn}^{01} &= \sum_{j=1}^n S_{mj}^{00}, \\ S_{mn}^{rs} &= \sum_{i=1}^m S_{in}^{r-1,s} = \sum_{j=1}^n S_{mj}^{r,s-1}, & m, n, r, s &= 1, 2, 3, \dots \end{aligned}$$

The double series is said to be summable  $(C, r, s)$  with summability value  $S$  if  $S_{mn}^{rs} r! s! / m^r n^s \rightarrow S$  as  $m, n \rightarrow \infty$ . It is bounded  $(C, r, s)$  if  $S_{mn}^{rs} r! s! / m^r n^s$  is bounded for all  $m$  and  $n$ . It is summable-bounded  $(C, r, s)$  if it is both summable  $(C, r, s)$  and bounded  $(C, r, s)$ . In case the  $u_{mn}$  are functions of complex variables  $x$  and  $y$  similar definitions can be set up for uniform summability, uniform boundedness, and uniform summability-boundedness  $(C, r, s)$ .

G. M. Merriman<sup>3</sup> has given the definition of summability  $(C, r, s)$  of a double series in a slightly different form. But our means differ from his only by a factor which is bounded and tends to one as  $m, n \rightarrow \infty$ . Consequently the two definitions are equivalent.

By means of Robison's<sup>4</sup> generalization of the Silverman-Toeplitz

<sup>1</sup> Bohr, H., *Bidrag til de Dirichlet'ske Raekkers Theori*, Dissertation, Copenhagen, 1910; *Über die Summabilität Dirichletscher Reihen*, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, 1909, pp. 247–262; *Sur la série de Dirichlet*, Comptes Rendus de l'Académie des Sciences, Paris, vol. 148 (1909), pp. 75–80.

<sup>2</sup> Throughout this paper  $m, n \rightarrow \infty$  means  $m$  and  $n$  tend to infinity simultaneously but independently.

<sup>3</sup> Merriman, G. M., *A set of necessary and sufficient conditions for the Cesàro summability of double series*, Annals of Mathematics, (2), vol. 29 (1928), pp. 343–354.

<sup>4</sup> Robison, G. M., *Divergent double sequences and series*, Transactions of this Society, vol. 28 (1926), pp. 50–73 (p. 53).