If an algebraic surface F of order  $\nu \ge 4$  has a double line (either nodal or cuspidal), the anomaly A of the plane sections C of a tangent cone to F is  $A = (1/6)\nu(\nu-4)(\nu-5)$ . As is evident from this expression for A, plane curve systems C associated with a surface having a double line are anomalous only for  $\nu \ge 6$ .

When  $\nu = 3$ , the above formula yields A = 1. However, a cubic surface with a nodal line is ruled, and a cubic surface with a cuspidal line is a cone. The treatment given in the two preceding sections and the resulting expression for A do not apply to such surfaces.

Wells College

## ON THE STABILITY OF THE LAMINAR FLOW OF A VISCOUS FLUID<sup>1</sup>

## RUDOLPH E. LANGER

The problem of the effect of two-dimensional first-order disturbances upon the laminar flow of an incompressible viscous fluid is known to be fundamental to the analysis of the phenomenon of turbulence. This discussion is concerned with such a problem in the case of a flow which takes place parallel to and between parallel plane boundaries. If the direction normal to these boundaries is designed by y, and that of the flow by x, the unit of length and the origin of coordinates may be chosen so that the boundary planes are given by the equations y=1 and y=-1. It is to be assumed then that the velocity of the undisturbed flow depends only on y, and is given by a function U(y), which is suitably differentiable and nonnegative, which is nonincreasing as to |y|, and which vanishes at the boundaries. If the maximum velocity is chosen as the unit, as will be assumed, it follows that

$$U(0) = 1,$$
  $U(1) = 0,$   $U(-y) \equiv U(y).$ 

The disturbance imposed upon this flow is to be taken as an elementary wave of length  $2\pi/\alpha$ , in the direction of flow.

The problem as stated is known<sup>2</sup> to admit of formulation as the differential boundary problem

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<sup>&</sup>lt;sup>1</sup> Presented to the Society, September 7, 1939.

<sup>&</sup>lt;sup>2</sup> C. L. Pekeris, On the stability problem in hydrodynamics, I, Proceedings of the Cambridge Philosophical Society, vol. 32 (1936), p. 55, and II, Journal of the Aeronautical Sciences, vol. 5 (1938), p. 236.