δ_{n+1} are to be chosen independently of $\delta_{n+2}, \delta_{n+3}, \cdots$ in such manner that the conjugate of the point w_n with respect to R lies exterior to the circle $|w| = 2^{n+1}$; each δ_n (for $n > 1$) is subjected then to two conditions, and the numbers δ_n can be determined in succession. The resulting region R is a Jordan region. The sequence w_n approaches the boundary point $w = 3$ of R, and the conjugate of w_n with respect to R becomes infinite with n. Theorem 3 is established.

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ON THE ORDER OF THE PARTIAL SUMS OF FOURIER POWER SERIES[1]

OTTO SZÁSZ

Dedicated to L. Fejér on his sixtieth birthday.

Let $f(x)$ be a Lebesgue integrable function, and denote the partial sums of its Fourier series by $s_n(f;\, x)$. It is well known that $s_n = o(n)$ uniformly[2] in x. Recently W. C. Randels[3] gave an example showing that this estimate cannot be improved. The same conclusion can be drawn from a note by E. C. Titchmarsh;[4] and A. Zygmund in his review of Randels' article (Zentralblatt für Mathematik, vol. 18, p. 353) pointed to another device, using convex coefficient sequences, which would establish the same fact.

In this note a simple construction is given, using a sequence of polynomials in the complex variable z. This leads to a sharper result showing that even for Fourier power series (that is, a power series considered on its circle of convergence and integrable) the estimate cannot be improved. Moreover, an example $F(z) = \sum_{n=0}^{\infty} c_n z^n$ is given which has the additional property that $F(z)/(1-z)$ is a generalized Fourier power series on $|z| = 1$.

We start with a sequence of polynomials of increasing degree $P_n(z) = (\sum_{\nu=0}^{m} c_{n\nu} z^\nu)^2 = \sum_{\nu=0}^{2m} a_{n\nu} z^\nu$ having the following properties:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |\, P_n(e^{ix}) \,|\, dx = \sum_{\nu=0}^{m} |\, c_{n\nu} |^2 = 1, \tag{1}$$

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[1] Presented to the Society, April 15, 1939.

[2] In fact, if $c_0, c_1, \cdots$ are the Fourier coefficients, then $c_n \to 0$. Hence $\sum_0^n |c_\nu| = o(n)$.

[3] W. C. Randels, *On the order of the partial sums of a Fourier series*, this Bulletin, vol. 44 (1938), pp. 286–288.

[4] E. C. Titchmarsh, *Principal value Fourier series*, Proceedings of the London Mathematical Society, (2), vol. 23 (1925), pp. xli–xliii.