FOURIER EXPANSIONS OF MODULAR FORMS AND PROBLEMS OF PARTITION[1]

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The subject which I am going to discuss in this lecture excels in the richness of its ramifications and in the diversity of its relations to other mathematical topics. I think therefore that it will serve our present purpose better not to attempt a systematic treatment, beginning with definitions and proceeding to lemmas, theorems, and proofs, but rather to look around and to envisage some outstanding marks scattered in various directions. I hope that the intrinsic relationships connecting the problems and theorems which I shall mention will nevertheless remain quite visible.

A good deal of the investigations about which I shall report can be subsumed under the heading of analytic number theory, and, more specifically, analytic additive number theory. It would, however, be a misplacement of emphasis if we were to look upon analysis, which here means function theory, only as a tool applied to the investigation of number theory. It is more the inner harmony of a system which I wish to depict, properties of functions revealing the nature of certain arithmetical facts, and properties of numbers having a bearing on the character of analytic functions.

Whereas the multiplicative number theory, which deals with questions of factorization, divisibility, prime numbers, and so on, goes back more than 2000 years to Euclid, the history of additive number theory, in any noteworthy sense, begins with Euler less than 200 years ago. In his famous treatise, *Introductio in Analysin Infinitorum* (1748), Euler devotes the sixteenth chapter, "De partitione numerorum," to problems of additive number theory. A "partition" is, after Euler, a decomposition of a natural number into summands which are natural numbers, for example, $6=1+1+4$. We can impose various restrictions on the summands; they may belong to a specified class of numbers, let us say odd numbers, or squares, or cubes, or primes; it may be required that they be all different; or their number may be preassigned. I wish to speak here only about unrestricted partitions. By the way, only the parts are essential, not their arrangement, so that we do not count two decompositions as different if they differ only in the order of the summands; we can therefore take the summands ordered according to their size.

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