A THEOREM ON SIMULTANEOUS REPRESENTATION OF PRIMES AND ITS COROLLARIES*

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1. **Simultaneous representation of primes.** Two numbers m and M are said to be represented simultaneously by a ternary form

$$f = ax^2 + by^2 + cz^2 + 2ryz + 2sxz + 2txy \tag{1}$$

and its reciprocal†

$$F = AX^2 + BY^2 + CZ^2 + 2RYZ + 2SXZ + 2TXY \tag{2}$$

if there exist integers x, y, z and X, Y, Z such that $f(x, y, z) = m$, $F(X, Y, Z) = M$ and $xX + yY + zZ = 0$.

The case of interest is that in which representation is not only simultaneous but also proper.‡ One is usually interested in the existence of such numbers m and M, fulfilling certain conditions, with the view of a suitable normalization of the given form f and its reciprocal F.§

In this paper we will require that m and M be a pair of simultaneously and properly represented distinct odd primes or doubles of such primes and derive a normalized form permitting some interesting applications. We note that the first coefficient a of f and the third coefficient C of F are represented simultaneously and properly and express our result as the following theorem.

THEOREM 1. *If f is a ternary quadratic form with a properly primitive reciprocal and if f is* (i) *properly or* (ii) *improperly primitive, then it is equivalent to a form f' such that* (i) *a' and C' are distinct odd primes not dividing $2\Omega\Delta$, or* (ii) *$a' = 2\alpha$ and α and C' are distinct odd primes not dividing $2\Omega\Delta$. Here a' is the leading coefficient of f', and C' is the third coefficient of the reciprocal F' of f'.*

We note that since F is properly primitive it represents properly an integer prime to any assigned integer and hence to $2\Omega\Delta$. If $\Omega\Delta$ is odd, then F represents properly an integer congruent to 1 (mod 4)

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† See Dickson, *Studies in the Theory of Numbers*, University of Chicago Press, p. 12.

‡ Ibid.

§ Dickson, ibid., pp. 15–17 and 54–60; P. Bachman, *Die Arithmetik der quadratischen Formen*, vol. 1, p. 64; H. J. S. Smith, *Collected Works*, vol. 1, pp. 455–509.