# ON FINITE ABELIAN $p$-GROUPS*

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Let $p$ denote a prime number which is fixed throughout the course of the paper. As we shall deal only with abelian groups whose order is some power of $p$, the term group will be used exclusively to designate a group of this nature.

DEFINITION. *A subgroup $\mathfrak{H}$ of a group $\mathfrak{G}$ is said to be reciprocally cyclic if the factor group $\mathfrak{G}/\mathfrak{H}$ is cyclic.*

The letters $k, m, n, r, x$ will be used to denote nonnegative integers and the symbol $(S_1, \cdots, S_n)$ to denote the group generated by the elements $S_1, \cdots, S_n$.

DEFINITION. *A set of elements $S_1, \cdots, S_n$ of the group $\mathfrak{G}$ is said to define a composition series for $\mathfrak{G}$ if the group series*

$$(S_1, \cdots, S_n), (S_1, \cdots, S_{n-1}), \cdots, (S_1, S_2), (S_1), (E)$$

*is a composition series for the group $\mathfrak{G}$.*

DEFINITION. *A set of elements $S_1, S_2, \cdots, S_n$ of the group $\mathfrak{G}$ is said to form a coverage system of the group $\mathfrak{G}$ if for any reciprocally cyclic subgroup $\mathfrak{H}$ of $\mathfrak{G}$ we may select elements $S_{m_1}, \cdots, S_{m_k}$ so that the group series*

$$(S_{m_1}, \cdots, S_{m_k}, \mathfrak{H}), (S_{m_1}, \cdots, S_{m_{k-1}}, \mathfrak{H}), \cdots, (S_{m_1}, \mathfrak{H}), \mathfrak{H}$$

*is a composition series from $\mathfrak{G}$ to $\mathfrak{H}$.*

A set of elements $S_1, \cdots, S_n$ which defines a composition series for a group $\mathfrak{G}$ also forms a coverage system for $\mathfrak{G}$. For let

$$(S_1, \cdots, S_n), (S_1, \cdots, S_{n-1}), \cdots, (S_1, S_2), (S_1), (E)$$

be the composition series of $\mathfrak{G}$ resulting from the element system $S_1, \cdots, S_n$ and $\mathfrak{H}$ any reciprocally cyclic subgroup of $\mathfrak{G}$; then the series

$$(S_1, \cdots, S_n, \mathfrak{H}), (S_1, \cdots, S_{n-1}, \mathfrak{H}), \cdots, (S_1, \mathfrak{H}), \mathfrak{H}$$

is a composition series from $\mathfrak{G}$ to $\mathfrak{H}$ with possible repetitions. These may be removed by starting with $S_1$ and striking out successively from left to right each member of the set $S_1, \cdots, S_n$ which is unnecessary for the generation of the groups of the above series. The

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