THE LAPLACE HEAVISIDE METHOD FOR BOUNDARY VALUE PROBLEMS*

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The usual methods for solution of boundary value problems for linear differential equations are based on taking linear combinations with "undetermined" coefficients of particular solutions of the homogeneous or nonhomogeneous equation. Subsidiary calculations are then required to deduce coefficient values such that the boundary conditions are satisfied. This note is concerned primarily with the formulation of a novel viewpoint in the Laplace integral treatment of differential equations and presents two new methods, neither involving undetermined coefficients, which are alternatives to the standard procedures.

Our developments stem from relations (α). The Laplace transform $F(p)$ plays the determining role. This function does not appear in the current developments of the definite integral solutions for variable coefficient equations. Since our methods yield solutions automatically satisfying prescribed boundary relations, they bear the same relationship to other procedures as does the Heaviside method, heretofore available for the one point problem for constant coefficient equations. Indeed our Method 1 is shown to be the natural generalization of the Heaviside procedure.

Consider, for real a, b,

$$(1) \qquad F(p) = \int_a^b y(x)e^{-xp}dx, \qquad -\infty < a < b < \infty.$$

Then, for $y(x)$ continuous,

(α1) $F(p)$ is entire and of exponential type,

(α2) $|F(p)| \leqq M\,(e^{-a\sigma}+e^{-b\sigma})/|\sigma|,\ \sigma=\Re(p)$.†

Furthermore

$$(1.1) \qquad y(x) = 1/2\pi i\int_{c-i\infty}^{c+i\infty} e^{xp}F(p)dp, \qquad c \text{ a real positive constant.}$$

For much of our work we shall be interested in the system‡

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† Extensions involving the Riemann-Lebesgue lemma or differentiability properties of $y(x)$ are unnecessary for our present purpose.

‡ The differential equation terminology in this article is consistent with that of Ince, *Ordinary Differential Equations.* This book is designated by I in subsequent references.