

SHORTER NOTICES

Valeur Pratique et Philosophie des Probabilités. By É. Borel. (Traité du Calcul des Probabilités et de ses Applications, vol. 4, no. 3.) Paris, Gauthier-Villars, 1939. 9+182 pp.

The treatise on probability and its applications, published by É. Borel and distinguished collaborators has been brought to a close by the present book, in which Borel discusses the place of the calculus of probability in man's analysis of the world about him. Borel does not attempt a systematic formal presentation: the book is an agreeably written non-technical discussion of various crucial aspects of the subject. In Borel's opinion, all our knowledge is probability knowledge, and his whole discussion has that point of view. Thus he even considers the problem of evaluating the probability that the sun will rise tomorrow, or the probability *a priori* that astrology is correct. It is difficult sometimes to determine whether probability, as Borel uses the term, refers to an individual's feelings, as measured by odds he might be willing to accept on a certain event, or whether probability refers to an event occurring in accordance with given specified conditions. Thus Borel considers a "reasonable person" *R* who is told that 12 horses are to race, who knows nothing about the horses, but who is nevertheless asked the probability that a certain horse *A* will win. According to Borel, *R* would try to avoid making any judgment in such a vaguely defined situation, but if pressed, could only give the answer $1/12$. It seems to the reviewer that if *R* knows nothing about the horses, he also knows nothing about the probabilities concerned, and that he must necessarily be silent on the whole situation. If the answer $1/12$ is proper why should *R* be reluctant to give it? The answer $1/12$ is not really an answer to the question. In fact the individual who answers $1/12$ has constructed a new problem: given that an event can occur in 12 ways, the conditions of occurrence being such that the theory of probability is applicable, and such that no one manner of occurrence is favored over another, what is the probability that the event will occur in way *A*? The answer is of course $1/12$; in fact the question was framed with that answer in mind. But the question is not the original one, because the relevant features of the frame of reference of the experiment have been made specific in the second version, while no useful hint of the real frame of reference was given in the first version.

At one point Borel makes a more or less formal definition. Probability is defined subjectively for an individual by the conditions of a bet he would accept. An objective probability value, to Borel, is a value which is the same "for a certain number of individuals equally well-informed on the conditions of the chance event." Since these betting odds are judged by the individuals concerned using their past experience and their desire to profit in the long run, their probability values can be checked by comparison with observed frequencies, which may suggest changed estimates, or even changes in the technique of estimating.

Estimating probabilities is thus the expected combination of experiment and rational thinking. There seems little point in arguing over definitions of probability as an empirical concept. The exact way probability is defined is of little importance as against the way the numbers when obtained are checked and used. The reviewer can see no possible use or meaning in a number which is called the probability that the sun will rise tomorrow. If one adopts the view that an event to which the probability calculus is applicable should have certain characteristics, leading to an actual