ON THE LOGARITHMIC SOLUTIONS OF THE GENERALIZED HYPERGEOMETRIC EQUATION WHEN $p=q+1$

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1. **Introduction.** In a recent paper,* the author gave the relations among the non-logarithmic solutions of the equation

$$\left\{ \prod_{t=1}^{q+1} (\theta + a_t) - \frac{1}{z} \prod_{t=1}^{q+1} (\theta + c_t - 1) \right\} y = 0, \tag{1}$$

where $\theta = z(d/dz)$ and where the a_t and c_t are any constants, real or complex, the only restriction being that one of the c_t must be equal to unity. Such solutions can be found in a number of places in the literature.† But in attempting to study the logarithmic cases of the problem treated in the above-mentioned paper, the author was unable to find the logarithmic solutions of equation (1) in the literature. It is the purpose of this paper to present these logarithmic solutions, but for the sake of completeness, the non-logarithmic solutions are also given. The methods used are those of Frobenius.‡

2. **Non-logarithmic solutions.** The solutions of equation (1) about the point $z=0$ are all non-logarithmic in character if no two of the c_t are equal or differ by an integer; and even if some of the c_t are equal or differ by an integer, the solutions will continue to be non-logarithmic provided certain of the c_t are equal to or differ from certain of the a_t by an integer. Since these special cases can easily be recognized, we shall avoid them in our theorems by making the hypotheses stronger than necessary.

THEOREM 1. *If no two of the c_t are equal or differ by an integer, then the solutions of equation* (1) *about the point $z=0$ are non-logarithmic in character and may be written in the form*

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* F. C. Smith, *Relations among the fundamental solutions of the generalized hypergeometric equation when $p=q+1$. I. Non-logarithmic cases*, this Bulletin, vol. 44 (1938), pp. 429–433.

† See, for example, L. Pochhammer, *Ueber die Differentialgleichung der allgemeineren hypergeometrischen Reihe mit zwei endlichen singulären Punkten*, Journal für die reine und angewandte Mathematik, vol. 102 (1888), pp. 76–159.

‡ G. Frobenius, *Ueber die Integration der linearen Differentialgleichungen durch Reihen*, Journal für die reine und angewandte Mathematik, vol. 76 (1873), pp. 214–235.