CREMONA INVOLUTIONS DETERMINED BY A PENCIL OF SURFACES*

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1. **Introduction.** The characteristics of the involutorial Cremona transformations determined by a pencil of surfaces of order n and containing an $(n-2)$-fold line d have been determined by Carroll [1]. The particular features of these transformations, which arise when the surfaces of the pencil are of order 3 and the curve residual to the line d in the base of the pencil is composite, have been considered in some detail by the same author [2]. Snyder [3] has suggested that a similar study of involutions defined by surfaces of higher order might be of interest.

The transformation is defined by Carroll as follows. Let

$$\lambda_1 F_n(x) - \lambda_2 F_n'(x) = 0 \tag{1}$$

be a pencil of surfaces of order n containing the line $d \equiv x_1 = 0,\ x_2 = 0$ to multiplicity $n-2$. Let $(z) = (0, 0, z_3, z_4)$ be a variable point on the line d, and let the pencil of surfaces (1) be connected with (z) by the relation

$$z_3\phi_1(\lambda_1, \lambda_2) - z_4\phi_2(\lambda_1, \lambda_2) = 0, \tag{2}$$

where ϕ_i, $(i=1, 2)$, is a binary form of order k. A point (y) of space determines a surface of the pencil (1) and hence a value of the ratio $\lambda_1:\lambda_2$, which in turn determines a point (z) of d. The line joining (y) and (z) meets the member of (1) determined by (y) in one further point (y'), the transform of (y) in the involution. The characteristics of the transformation are

$$\begin{aligned}
S_1 &\sim S_{2n(k+1)-1}: d^{2(n-2)(k+1)}k(n-2)\bar{d}^2\\
&\qquad C_{4n-4}^{2k+1}\{(6n-8)k+6n-10\}g,\\
d &\sim T_{2n(k+1)-2}: d^{2(n-2)(k+1)}k(n-2)\bar{d}\\
&\qquad C_{4n-4}^{2k+1}\{(6n-8)k+6n-10\}g,\\
C_{4n-4} &\sim \Sigma_{4n(k+1)-4}: d^{4(n-2)(k+1)}\ k(n-2)\bar{d}^4\\
&\qquad C_{4n-4}^{4k+1}\{(6n-8)k+6n-10\}g^2,\\
R_{nk+2n-1} &\sim R_{nk+2n-1}: d^{(n-2)(k+2)}\ k(n-2)\bar{d}\\
&\qquad C_{4n-4}^{k+1}\{(6n-8)k+6n-10\}g,
\end{aligned} \tag{3}$$

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