

Using Lemma 3 we have

$$\begin{aligned} S_n^{(k-1)} &= \sum_{j=0}^{k-1} \frac{(-1)^{j+n} C_{n+k, j}}{2^k} \Delta^j a_0 + C_{n+k, k-1} \sum_{j=0}^{k-1} \frac{\Delta^j a_0}{2^{j+1}} + O(n^{k-2}) \\ &= \frac{(-1)^{k+n-1} C_{n+k, k-1}}{2^k} \Delta^{k-1} a_0 + C_{n+k, k-1} \sum_{j=0}^{k-1} \frac{\Delta^j a_0}{2^{j+1}} + O(n^{k-2}). \end{aligned}$$

Then

$$\lim_{n \rightarrow \infty} c_n^{(k-1)} = \lim_{n \rightarrow \infty} \frac{S_n^{(k-1)}}{C_{n+k-1, k-1}} = \frac{\Delta^{k-1} a_0}{2^k} \lim_{n \rightarrow \infty} (-1)^{k+n-1} + \sum_{j=0}^{k-1} \frac{\Delta^j a_0}{2^{j+1}}.$$

This limit fails to exist. Consequently, under the hypotheses of our theorem, the series $\sum_{n=0}^{\infty} (-1)^n a_n$ is not summable $(C, k-1)$.

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A GENERAL CONTINUED FRACTION EXPANSION*

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Introduction. Considerable attention has been given at various times by many writers to the function-theoretic character of continued fractions of the form

$$1 + \frac{a_1 x}{1} + \frac{a_2 x}{1} + \dots$$

Only a very restricted class of power series, the "seminormal" ones, admit an expansion into a continued fraction of this type (cf. Perron [3, p. 301]). For example, the power series expansion about the origin of the function $1+x^2$ fails to be seminormal. In §1 of this paper we show that *every* power series admits an expansion into a continued fraction of a form which is a generalization of that above. Many of the older theorems have immediate generalizations. These are presented without proof when the demonstration parallels that for the seminormal case.

In §2 we discuss the question of gaps in seminormal power series. In §3 an important special case is considered.

1. Expansions in continued fractions. Let

$$(1.1) \quad 1 + \frac{a_1 x^{\alpha_1}}{1} + \frac{a_2 x^{\alpha_2}}{1} + \dots$$

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