The method here described is based on the suggestions made by Mr. Frederick King. These suggestions have led to the evaluation of $R(x)$ as a starting point of the subsequent discussion.

NEW YORK CITY

ALL INTEGERS EXCEPT 23 AND 239 ARE SUMS OF EIGHT CUBES

L. E. DICKSON

Summary. In 1770 Waring stated that every positive integer is a sum of nine integral nonnegative cubes. The first proof is due to Wieferich.* I shall prove the following new result.

THEOREM. *Every positive integer other than* 23 *and* 239 *is a sum of eight integral nonnegative cubes.*

Five lemmas are required.

LEMMA 1. *Every integer greater than or equal to* $233^6 D$ *is a sum of eight cubes if* $D = 14.0029682$, *or more generally if* $D = d$, *where*†

$$d > 14 + \left(\frac{24}{167}\right)^3, \qquad d \leqq 14.1.$$

The algebraic part of Wieferich's proof holds for all integers exceeding $2\frac{1}{4}$ billion. The fact that all smaller integers are sums of nine cubes was proved by use of Table I. To prove my theorem, I shall need also the new Tables II and III.

Table I gives, for each positive integer $N \leqq 40{,}000$, the least number m such that N is a sum of m cubes.

It was computed by R. D. von Sterneck‡ by adding all cubes to

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* His errors are avoided in the much simpler proof by the writer, Transactions of this Society, vol. 30 (1928), pp. 1–18. On page 16 is proved a generalization of Landau's result that all sufficiently large numbers are sums of eight cubes.

† The proof is essentially like that given for $d = 14.1$ by W. S. Baer, *Beiträge zum Waringschen Problem*, Dissertation, Göttingen, 1913.

‡ Sitzungsberichte der Akademie der Wissenschaften, Vienna, IIa, vol. 112 (1903), pp. 1627–1666.