

SOME REMARKS ON LINEAR DIFFERENTIAL SYSTEMS*

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1. **Introduction.** In 1926 Bliss[†] formulated a definition of self-adjoint and definitely self-adjoint systems of ordinary linear differential equations of the first order with two-point boundary conditions, the coefficients of the system being real. More recently, Bliss[‡] has modified the definition of definite self-adjointness in such a manner that the boundary value problems arising in a well known way from nonsingular problems of Bolza in the calculus of variations are definitely self-adjoint by the new definition. Further intimate relationships that exist between such definitely self-adjoint boundary value problems and the boundary value problems associated with the calculus of variations have been established by Reid.[§]

In this note linear differential systems with complex-valued coefficients are considered and self-conjugate adjoint and definitely self-conjugate adjoint systems are defined. It is shown that definitely self-conjugate adjoint systems have only real characteristic values, and that for a consideration of the questions of the existence of characteristic values and associated expansion theorems such a system may be replaced by a corresponding definitely self-adjoint system with real coefficients. In §4 there is solved an associated matrix differential equation, and the question of self-conjugate adjointness is reduced to the determination of a nonsingular constant matrix of a certain type. Finally, there is given an application of this matrix differential equation to the theory of matrix differential equations of Riccati type.

Matrix notation is used throughout. Capital italic letters denote n -rowed square matrices whose elements are complex-valued, the element in the i th row and j th column being denoted by the same letter with the subscript ij . Lower case italic letters signify vectors with n components, the i th component being denoted by a subscript i . If $M \equiv \|M_{ij}\|$, $u \equiv [u_i]$, the vectors $[M_{ij}u_j]$ and $[u_i M_{ji}]$ are denoted by Mu and uM , respectively, where the repetition of the subscript j indi-

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† Transactions of this Society, vol. 28 (1926), pp. 561–584. This paper will be referred to as B.I.

‡ Transactions of this Society, vol. 44 (1938), pp. 413–428. This paper will be referred to as B.II.

§ Transactions of this Society, vol. 44 (1938), pp. 508–520.