ON GREEN'S FUNCTIONS IN THE THEORY OF HEAT CONDUCTION*

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1. **Introduction.** In this Bulletin (vol. 44 (1938), p. 125) Lowan discusses the Green's function for a line source at (r', θ') for the cases where the solid (i) is an infinite cylinder $r=a$, and (ii) is bounded internally by $r=a$, radiation taking place at $r=a$ into a medium at zero temperature.

He uses the method of the Laplace transformation. The solution for the first problem agrees with that already obtained by contour integration.† There are some obvious misprints in his discussion of the second problem, and in his solution on page 133 he seems to have used as boundary condition $G=0$ on $r=a$, instead of $\partial G/\partial r+hG=0$, in his notation. His result on that page should read

$$G = u + v = \frac{1}{4\pi}\sum_{n=-\infty}^{\infty}\cos n(\theta-\theta_0)\int_{-\infty}^{\infty}\alpha e^{-k\alpha^2 t}\frac{H_n^{(1)}(\alpha r_0)}{U_n(\alpha a)}$$

$$\cdot\left\{J_n(\alpha r)U_n(\alpha a) - H_n^{(1)}(\alpha r)\left[\alpha\frac{d}{dz}J_n(z) + hJ_n(z)\right]_{z=\alpha a}\right\}d\alpha,$$

where

$$U_n(\alpha a) = \left[\alpha\frac{d}{dz}H_n^{(1)}(z) + hH_n^{(1)}(z)\right]_{z=\alpha a}.$$

Put in this form it can be reduced to the simpler form given below in (16), except for the difference in the sign of h.

In this paper we discuss this second problem, first by contour integration, and second by the Laplace transform. In the latter we use what appears to us a much simpler notation and a more rapid approach to the solution.

We remark also that we have used this notation and method in a number of other questions‡ and believe that it will be found increasingly useful and much simpler than the operational methods,

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† Carslaw, *Conduction of Heat*, 2d edition, 1921, §§88, 89. This book will be referred to below as C.H.

‡ Cf. Carslaw, *Operational methods in mathematical physics*, Mathematical Gazette, vol. 22 (1938), pp. 264–280; Carslaw and Jaeger, *Some problems in the mathematical theory of the conduction of heat*, Philosophical Magazine, (7), vol. 26 (1938), pp. 473–495.