JACKSON SUMMATION OF THE FABER DEVELOPMENT*

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1. **Introduction.** The purpose of this note is to prove the following theorem:

THEOREM. *Let C be an analytic Jordan curve in the z-plane, and let $f(z)$ be analytic in C, continuous in $\overline{C}$, the closed limited set bounded by C, and let*† $f^{(p)}(z)$, $(p \geqq 0)$, *satisfy a Lipschitz condition*‡ *of order* α, $(0 < \alpha \leqq 1)$, *on C. Then*

$$\left| f(z) - \sum_{\nu=0}^{n} d_{n\nu} a_\nu P_\nu(z) \right| \leqq \frac{M}{n^{p+\alpha}}, \qquad z \text{ in } \overline{C}, \tag{1}$$

where M is a constant independent of n and z,

$$\sum_{\nu=0}^{n} a_\nu P_\nu(z)$$

is the sum of the first $n+1$ terms of the development of $f(z)$ in the Faber§ *polynomials belonging to C, and $d_{n\nu}$ is the Jackson*|| *summation coefficient of order p.*

In a previous paper the author¶ showed that under the above hypothesis

$$\left| f(z) - \sum_{0}^{n} a_\nu P_\nu(z) \right| \leqq M(\log n / n^{p+\alpha}), \qquad z \text{ in } \overline{C}.$$

Later John Curtiss** proved the existence of a sequence of polynomials $Q_n(z)$ of respective degrees n, $(n = 1, 2, \cdots)$, such that

$$| f(z) - Q_n(z) | \leqq M/n^{p+\alpha}.$$

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† $f^{(p)}(z)$ denotes the pth derivative of $f(z)$; $f^{(0)}(z) \equiv f(z)$.

‡ $f(z)$ satisfies a Lipschitz condition of order α on C if for z_1 and z_2 arbitrary points on C we have $|f(z_1) - f(z_2)| \leqq L|z_1 - z_2|^\alpha$, where L is a constant independent of z_1 and z_2.

§ G. Faber, Mathematische Annalen, vol. 57 (1903), pp. 389–408.

|| Dunham Jackson, Transactions of this Society, vol. 15 (1914), pp. 439–466; p. 463.

¶ This Bulletin, vol. 41 (1935), pp. 111–117; this paper will be referred to hereafter as SI.

** This Bulletin, vol. 42 (1936), pp. 873–878.