ON CONTIGUOUS POINT SPACES

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In this paper we shall briefly indicate the kind of geometry obtained by a modification of one of Hausdorff's axioms for topological space. The resulting space turns out to be the contiguous point space of R. L. Moore (Rice Institute Pamphlet, vol. 23 (1936)). However, unlike Moore, we shall define *contiguity*, and we shall define it in terms of *point* and *neighborhood*. It is in terms of these two primitive indefinables that Hausdorff states his axioms for topological space (*Mengenlehre*, p. 228):

Axiom 1. *Every point p has a neighborhood U_p. For every p, $p \in U_p$.*

Axiom 2. *For every two neighborhoods U_p and V_p of the same point, there is a third $W_p \subset U_p \cdot V_p$.*

Axiom 3. *Every point $q \in U_p$ has a neighborhood $U_q \subset U_p$.*

Axiom 4. *For every p and q, $p \neq q$ implies that there exist neighborhoods U_p and U_q such that $U_p \cdot U_q = 0$.*

A contiguous point space will be defined by the Axioms 1–3 together with the following new axiom:

Axiom 4′. *There exist points, for example, p and q, such that $p \neq q$ and such that for every U_p and U_q the common part $U_p \cdot U_q$ contains both p and q.*

This axiom is obtained by negating 4 and substituting the condition $U_p \cdot U_q \supset (p, q)$ for the weaker condition $U_p \cdot U_q \neq 0$. The property given by 4′ approximates our ordinary idea of contiguity; we set this down as a formal definition.

Definition. *The point p is said to be contiguous to the point q if* (1) *$p \neq q$, and* (2) *any neighborhood of the one point contains the other.*

First, it may be pointed out, no space containing contiguous points can be a topological space. This is obvious from the method of deriving 4′. In topological space a set must have at least a denumerable infinity of points in order for it to have a limit point. This is not true for contiguous points since, if p and q are contiguous, the point p is a limit point of the set (q), which is a set containing only one point.

Theorem 1. *No point is contiguous to itself.*