METRIC SPACES WITH GEODESIC RICCI CURVES. II.

JACK LEVINE

1. **Introduction.** In this paper we give a partial classification of four-dimensional metric spaces admitting geodesic Ricci curves. The results of I* will be assumed known, along with the notations of that paper.

We assume a given set of independent vectors† $\lambda^{i}_{a|}$ such that $c^{i}_{ij}=0$ (i not summed), so as to obtain geodesic curves, and impose conditions (24), (25), (26) of I on the θ_a. From I, (25) we see that if $\mu_{kk}=0$ for any k, then by I, (24), we have (since $|\lambda^{i}_{a|}|\neq 0$) $\partial\mu_k/\partial x^a=0$, or $\mu_k=\text{const}$. If, however, for any given k, $\mu_{kk}\neq 0$, then from I, (25), $c^{k}_{ij}=0$, for all i and j. This gives us a means of classifying the spaces according to the number of μ_{kk} which equal zero. For $n=4$ there are five cases, which, without loss of generality, we may take in the form:

(A) $\mu_{ii}\neq 0$; (B) $\mu_{11}=0$; $\mu_{22}, \mu_{33}, \mu_{44}\neq 0$;

(C) $\mu_{11}=\mu_{22}=0$; $\mu_{33}, \mu_{44}\neq 0$; (D) $\mu_{11}=\mu_{22}=\mu_{33}=0$; $\mu_{44}\neq 0$;

(E) $\mu_{ii}=0$.

In the following discussion we consider cases (A), (B), and certain special cases under (C). For these special cases we shall merely state the results.

2. **Cases** (A) **and** (B). For case (A) we see from I, (25) that $c^{k}_{ij}=0$, which implies that V_4 is a flat space.

We consider now case (B). Here μ_1 and hence θ_1 is constant. From I, (25) we have

$$\overset{2}{c}_{ij}=\overset{3}{c}_{ij}=\overset{4}{c}_{ij}=0. \tag{1}$$

If in I, (26) we make the substitution

$$\bar{c}^{\,i}_{jk}=\frac{\theta_j\theta_k}{\theta_i}\,c^{\,i}_{jk}, \tag{2}$$

we obtain I, (23) in the barred quantities. We call this resulting equation I, (23′).

* *Metric spaces with geodesic Ricci curves*, I, this Bulletin, vol. 44 (1938), pp. 145–152. We refer to this paper as I, and the notation I, (23), for example, refers to its equation (23).

† All indices take the values 1, 2, 3, 4 unless otherwise noted.