

**ON GENERATING THE SIMPLE GROUP  $LF(2, 2^N)$   
BY TWO OPERATORS OF PERIODS  
TWO AND THREE\***

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The purpose of this paper is to consider the question of the number of abstractly distinct ways in which it is possible for two operators of periods two and three to generate the simple group  $LF(2, 2^N)$ . The general procedure to be followed in studying such a problem has been outlined by Professor Brahana† and has already been applied to  $LF(2, 2^3)$  by the author.‡ Those previous results suggested the present generalization.

Since the  $2^{2N} - 1$  substitutions of period two in  $LF(2, 2^N)$  are all conjugate,§ it is sufficient in seeking possible generating operators to consider only one of them, say

$$T \equiv \begin{pmatrix} 1, 1 \\ 0, 1 \end{pmatrix}.$$

In the representation of  $G$  on  $2^N + 1$  letters,  $T$  leaves fixed the single element  $\infty$ . The largest subgroup within which  $T$  is invariant is  $G^{(\infty)}$ , composed of all the substitutions

$$T_\mu \equiv \begin{pmatrix} 1, \mu \\ 0, 1 \end{pmatrix}, \quad \mu \text{ in the } GF[2^N],$$

which keep the single element  $\infty$  unchanged. The group  $G^{(\infty)}$  is abelian, of order  $2^N$  and of type  $(1, 1, 1, \dots)$ .

If the operators of period three are divided up into sets of complete conjugates under  $G^{(\infty)}$ , then the various members of each set satisfy with  $T$  the same abstract relations, and it is sufficient to select from each such set only one operator to serve as a possible second generator with  $T$ . The number of these sets to be considered depends

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† H. R. Brahana, *Pairs of generators for the known simple groups whose orders are less than one million*, Annals of Mathematics, vol. 31 (1930), pp. 542, 543.

‡ *Necessary and sufficient conditions for generating certain simple groups by two operators of periods two and three*, American Journal of Mathematics, vol. 49 (1937), pp. 69–72, hereafter referred to as *Necessary and sufficient conditions*.

§ The various properties of  $LF(2, 2^N)$  of which use is made in this paper are given in Dickson's *Linear Groups with an Exposition of the Galois Field Theory*, 1901, chap. 12.