Einführung in die mathematische Logik und in die Methodologie der Mathematik. By Alfred Tarski. Vienna, Springer, 1937. 10+166 pp.

This book is an introduction in the strictest sense; the reader is led just inside one field after another, given a glimpse of interesting things in the interior, and dragged away. It is thoroughly elementary, written for a layman who cannot even be depended upon to know that "*w. z. b. w.*" means "*was zu beweisen war.*" But it is a rare combination of elementariness and competence, and experts can profit from the clear and correct formulation of elementary concepts which the book provides. Particularly noteworthy is the author's care in avoiding the common confusions between sign and object, or between use and mention of expressions.

The book begins with an explanation of the variable, a discussion of definition, and an introduction to the truth functions. Various principles of the truth-function calculus are made clear without recourse to truth tables or even symbolic notation. Then the basic principles of the theory of identity are explained; also the elementary concepts of class theory, stopping short of types; also the principal classifications of relations. Special attention is accorded to functions, which are explained as one-many relations. The method of defining arithmetical concepts in terms of logic is then roughly sketched. Part I concludes with an explanation of the methodological concepts of axiom, formal system, model, consistency, completeness, and independence. The latter three concepts are explained only in their traditional forms (as opposed to Post's formulations), so that they apply only to systems which presuppose elementary logic.

In Part II, three axiom systems for part of arithmetic are compared with a view to illustrating mathematical methodology The book closes with two comprehensive systems for real-number theory. In the course of these developments the reader becomes acquainted with such concepts as group, field, closed system, compactness, and continuity. The excellent exposition of the book is supplemented with an abundance of skillfully devised exercises.

It is only in criticizing this book by its own high standards that a few imperfections of formulation are to be found. Tarski distinguishes sharply between terms (*Bezeichnungen*) and statements (*Sätze*), and explains that whereas the variables "x," "y," $\cdots$, supplant terms, the variables "p," "q," $\cdots$, supplant statements. This is excellent, and calculated to eliminate questions which arise from the confused use of "p," "q," $\cdots$, as nouns: the question "What sort of objects do the variables 'p,' 'q,' $\cdots$ *denote*, or take as *values*?" or the question "What conditions are necessary in order that $p=q$?" But at a few points Tarski himself slips back into the use of "p" and "q" as nouns; thus we read "*Für beliebige p und q* $\cdots$," and again "*Wenn q aus p folgt und* $\cdots$."

Further, it would have been more in the spirit of his careful distinctions to have suppressed the common but confusing terminology "*Implikation*" and "*Equivalenz*" in favor of "conditional" ("*Bedingungssatz*") and "biconditional" ("*gegenseitiger Bedingungssatz*"). The conditional and biconditional are most naturally construed as modes of statement-composition ("if-then," "if and only if") coördinate with alternation ("or") and conjunction ("and"), whereas implication and equivalence are more naturally construed as metalogical relations between statements, expressed by inserting verbs ("implies," "is equivalent to") between names of the statements.

The reader should be warned, finally, against identifying Tarski's *Satzfunktion* with the propositional function of *Principia Mathematica.* The former is not a function at all, in the mathematical sense of a one-many relation, but is rather a *statement form* or *matrix*, an expression derived from a statement by putting variables