ON COMPLETELY CONTINUOUS LINEAR TRANSFORMATIONS*

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We shall deal with complete linear vector or so-called Banach spaces.† A completely continuous linear transformation is defined as a linear transformation which carries every bounded set into a compact set. In spaces of a finite number of dimensions, that is, spaces which are linear closed extensions of a finite number of elements, all bounded sets are compact.‡ Therefore singular transformations, that is to say, linear limited transformations which transform their domains into spaces of a finite number of dimensions, are completely continuous linear transformations. It is well known that the strong limit, or limit in the norm sense, of a sequence of completely continuous linear transformations is also completely continuous and linear.§ Consequently, the strong limit of a sequence of singular transformations is completely continuous and linear. The question naturally arises whether, conversely, every completely continuous linear transformation is the strong limit of a sequence of singular transformations.|| This paper obtains a result for the domain of the transformation, a Banach space, and the range, a space to be defined and hereafter to be referred to as of type A. It will be seen that the conception of a space of type A is really a generalization of the idea of a Banach space with a denumerable base,¶ which will hereafter be referred to as of type S.

By a space of type A we shall mean a Banach space in which there exists a linearly independent sequence $\{f_n\}$ of elements of unit norm and a double sequence $\{L_{mn}(g)\}$ of linear limited operators such that for every g

$$\lim_{m=\infty} \left\| g - \sum_{n=1}^{m_n} L_{mn}(g) f_n \right\| = 0. \tag{1}$$

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† Banach, *Théorie des Opérations Linéaires*, p. 53.

‡ Riesz, Acta Mathematica, vol. 41 (1927), p. 77.

§ Banach, loc. cit., p. 96.

|| Hildebrandt, this Bulletin, vol. 37 (1931), p. 196.

¶ By a space of type S we shall mean a Banach space with a finite or denumerably infinite set of elements $\{f_i\}$ of unit norm such that every element g may be uniquely represented in the form $g = \sum_{i=1}^{\infty} c_i(g) f_i$, or $\lim_{n=\infty} \| g - \sum_{i=1}^{n} c_i(g) f_i \| = 0$, where for a fixed index i the coefficients $c_i(g)$ are bounded linear operators on the space. See Schauder, Mathematische Zeitschrift, vol. 26 (1927), p. 47, and Banach, loc. cit., p. 110.