THE CONSTRUCTIVE SECOND NUMBER CLASS*

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The existence of at least a vague distinction between what I shall call the constructive and the non-constructive ordinals of the second number class, that is, between the ordinals which can in some sense be effectively built up to step by step from below and those for which this cannot be done (although there may be existence proofs), is, I believe, somewhat generally recognized. My purpose here is to propose an exact definition of this distinction and of the related distinction between constructive and non-constructive functions of ordinals in the second number class; where, again to speak vaguely, a function is constructive if there is a rule by which, whenever a value of the independent variable (or a set of values of the independent variables) is effectively given, the corresponding value of the dependent variable can be effectively obtained, effectiveness in the case of ordinals of the second number class being understood to refer to a step by step process of building up to the ordinal from below.

Much of the interest of the proposed definition lies, of course, in its absoluteness, and would be lost if it could be shown that it was in any essential sense relative to a particular scheme of notation or a particular formal system of logic. It is my present belief that the definition is absolute in this way—towards those who do not find this convincing the definition may perhaps be allowed to stand as a challenge, to find either a less inclusive definition which cannot be shown to exclude some ordinal which ought reasonably to be allowed as constructive, or a more inclusive definition which cannot be shown to include some ordinal of the second class which cannot be seen to be constructive.

It is believed that the distinction which it is proposed to develop between constructive and non-constructive ordinals (and functions of ordinals) should be of interest generally in connection with applications of the transfinite ordinals to mathematical problems. The relevance of the distinction is especially clear, however, in the case of applications of the ordinals to certain questions of symbolic logic (for example, various questions more or less closely related to the well known theorem of Gödel on undecidable propositions)†—this is

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† Kurt Gödel, *Über formal unentscheidbare Sätze der Principia Mathematica und*