*Les Fonctions Polyharmoniques*, by Miron Nicolesco. (Actualités Scientifiques et Industrielles, no. 331; exposés sur la théorie des fonctions, IV.) Paris, Hermann, 1936.

This monograph is the fourth in a series devoted to recent developments in the theory of functions, which is being published under the editorship of Professor Montel of Paris. These monographs have as an objective the presentation of synthetic summaries of some of the advances made during the last few years; no attempt is made at giving a detailed exposition, but a clear-cut sketch of the developments, supplemented by suitable bibliographies, is made available to the reader who wishes to orient himself in the fields treated.

The polyharmonic functions of order $p$ (or $p$-harmonic), in an $n$-dimensional space, are those functions $U(x_1, x_2, \cdots, x_n)$ of $n$ independent variables which satisfy the partial differential equation $\Delta^p U=0$, where $\Delta^p$ is the $p$th iteration of the Laplacian operator $\Delta$. Thus,

$$\Delta \equiv \sum_{s=1}^{n} \frac{\partial^2}{\partial x_s^2}, \ \Delta^k = \Delta(\Delta^{k-1}), \ (k = 1, 2, 3, \cdots), \ \Delta^0 \equiv 1.$$

For $p=1$, we have the case of the ordinary harmonic functions. The biharmonic case ($p=2$), being of importance in the theory of elasticity, was the object of research rather early. Thus, Airy introduced the stress function from which the components of the stress tensor may easily be calculated, and showed that it had to satisfy the equation $\Delta^2\phi=0$, with appropriate boundary conditions.

The earlier researches on polyharmonic functions centered, quite naturally, about the study of boundary value problems analogous to the classical ones in the theory of harmonic functions. Most of the results obtained in this direction are due to the Italian school of mathematicians led by Almansi, Volterra, Marcolongo, Lauricella, Boggio, and others. In these researches, the investigation of the intrinsic, structural properties of the polyharmonic functions was, for the most part, subsidiary to the solution of the particular boundary value problems being studied. Even so important a result as Almansi's expansion theorem was treated primarily from the point of view of its bearing on this type of problem.

Since 1930, Professor Nicolesco and others have undertaken a systematic study of the intrinsic properties of the polyharmonic functions for their own sake and have succeeded in filling important gaps in the theory of these interesting functions. The main purpose of his monograph is to give a synthesis of the results so far obtained. The work also contains a brief summary of some of the work done in connection with these functions and their associated boundary value problems, placing some emphasis on the biharmonic case of elasticity theory.

An extensive memoir by Professor Nicolesco, entitled *Recherches sur les fonctions polyharmoniques*, to appear shortly in the Annales de l'École Normale Supérieure, as a supplement to the present work, should be of the greatest interest.

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*Les Conditions de Monogénéité*. By D. Menchoff. (Actualités Scientifiques et Industrielles, no. 329.) Paris, Hermann, 1936. 53 pp.

This monograph, as its title implies, is a report on some of the recent developments in the direction of obtaining sufficient conditions that a function of a complex variable be monogenic at a point or holomorphic in a region. Monogeneity at a point being equivalent to the existence of a finite-valued derivative at the point, the initial