

the pairs of *distinct* points $b_0, b_1; c, b_0; b_1, c$ in turn. We obtain the result that ab_0b_1 or b_1b_0b and acb_0 or b_0cb and ab_1c or cb_1b exist. But it is readily verified that no one of the eight possible combinations can exist, for assuming any one of them leads, by an application of the transitive property of the betweenness relation in metric spaces, to the conclusion that the triple b_0, c, b_1 is linear, in violation of the relations (1) above.* This completes the proof of the theorem.

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A TRANSFORMATION ASSOCIATED WITH THE TRISECANTS OF A RATIONAL TWISTED QUINTIC CURVE†

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1. *Introduction.* This transformation is generated by the use of a (1, 1) correspondence between a pencil of ruled cubic surfaces $|F|$, and their simple directrices which are the trisecants of a rational twisted quintic curve C_5 . All of the cubic surfaces contain C_5 and have the quadrisecant of C_5 as a double line l . Through a general point P of space passes one F whose simple directrix r determines with P a plane tangent to the ruled surface of trisecants of C_5 at a point Q on r . The line PQ meets F in a residual point P' which is the image of P in an involutorial Cremona transformation of order 43. A special feature of the transformation is the existence of two ruled surfaces whose generators are parasitic lines of the transformation. One of these surfaces is a principal surface, and the other is not.

Other transformations generated by a somewhat similar method have been discussed by the author in recent papers.‡

2. *The Pencil of Cubic Surfaces.* The equation of a pencil of cubic surfaces, parameter λ , with a double line $l \equiv x_1 = x_2 = 0$, is

* *Erste Untersuchung*, loc. cit., p. 78. For example, the combination ab_0b_1, acb_0, ab_1c cannot exist since the second and third of these relations imply the existence of b_1cb_0 .

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‡ This Bulletin, vol. 42 (1936), pp. 535-540. Tôhoku Mathematical Journal, vol. 43, (1937), pp. 174-177.