REAL CANONICAL BINARY SYMMETRIC TRILINEAR FORMS*

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1. *Introduction.* In treating the equivalence of binary forms of the third degree under non-singular linear transformations it is natural to divide the problem into three parts. These are concerned respectively with the equivalence of forms of the type

$$a_{ijk}x_iy_jz_k,$$

where

(a) the vectors (x_1, x_2), (y_1, y_2), (z_1, z_2) are distinct;

(b) 2 of the vectors (x_1, x_2), (y_1, y_2), (z_1, z_2) are equal but not equal to the 3rd;

(c) the vectors (x_1, x_2), (y_1, y_2), (z_1, z_2) are equal.

Dedekind, Schwartz, others,† and the author have treated the problem of the equivalence of binary trilinear forms in the complex field. Specifically the problem solved was that of determining the conditions on (a_{ijk}), (a'_{pqr}) such that the form $T=a_{ijk}x_iy_jz_k$ be reducible to the given form $T'=a'_{pqr}x_p'y_q'z_r'$ under the linear transformations

$$(1) \qquad x_i = a_{ip}x_p', \qquad y_j = b_{jq}y_q', \qquad z_k = c_{kr}z_r',$$

where $i, j, k, p, q, r=1, 2$, and the matrices (a_{ip}), (b_{jq}), (c_{kr}) are non-singular. Recently the author solved this problem of equivalence for the field of reals.‡ The theory for case (a) in the field of reals and complexes is hence complete. In the present paper the author treats the equivalence of forms satisfying (b) for the field of reals.

Evidently, the *class of forms* (c) *is contained in the class* (b), which in turn is contained in the class (a).

In a new paper, to appear elsewhere, the author has solved the problem of equivalence of the forms in class (c), making

* Presented to the Society, March 26, 1937.

† For a complete list of references see R. Oldenburger, *On canonical binary trilinear forms*, this Bulletin, vol. 38 (1932), p. 385.

‡ R. Oldenburger, *Real canonical binary trilinear forms*, American Journal of Mathematics, vol. 59 (1937), pp. 427–435.