NOTE ON THE CONTINUITY OF THE ERGODIC FUNCTION

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1. *Introduction.* Let M denote a bounded point set and ϵ an arbitrarily chosen positive number. A continuous curve C is termed *ϵ-ergodic to M* if an arbitrary point of M lies at a distance $\leqq \epsilon$ from some point of C. Recently* I have shown that the set of continuous, rectifiable curves ϵ-ergodic to M contains a member whose length furnishes an absolute minimum for the lengths of the curves in the set. This member was called an *ergodic curve of M* and its length the *ergodic function* $\Lambda(\epsilon)$ *of M*. The function $\Lambda(\epsilon)$ is finite and non-negative, being equal to zero if and only if $\epsilon \geqq \rho$, where ρ is the radius of the smallest circular region containing M. In addition $\Lambda(\epsilon)$ was proved to be a monotone non-increasing function of ϵ which is always continuous on the right.

In this note it is shown that $\Lambda(\epsilon)$ is also continuous on the left (and is therefore continuous in the ordinary sense). In the original version of this paper I was able to prove this result only for a value $\epsilon_0(<\rho)$ of ϵ for which the set M had an ergodic curve which was an "ordinary curve" (a continuous curve which is either of class C' or else made up of a finite number of arcs of class C'). The general result announced above is made possible by Lemma 2 below for which I am indebted to Professor von Neumann.

2. *Preliminary Lemmas.* In this section we shall assemble a number of lemmas leading to the proof of the result announced in the introduction.

LEMMA 1. *The set M_1 of points lying at a distance $\leqq \epsilon$ from the points of a continuous rectifiable arc of length $2s$ joining two points A and B situated a distance $2c (c \leqq s)$ apart lies in a region composed of the points interior to two circles described about A and B as centers with radii equal to $\epsilon + (2^{1/2}\alpha + s/(2\epsilon))s$, where $\alpha^2 = 1 - c/s$.*

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* *Ergodic curves*, American Journal of Mathematics, vol. 58 (1936), pp. 727–734.