ON LOCALLY COMPACT METRISABLE SPACES

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The object of this paper is to present two characterizations, one of the class of locally compact metrisable spaces, the other of the class of locally compact, separable, metrisable spaces (locally compact, perfectly separable Hausdorff spaces), as follows.

THEOREM 1. *In order that a metrisable space be locally compact it is necessary and sufficient that it be the difference of two closed sets in every metric space in which it is topologically imbedded.*†

THEOREM 2. *In order that a Hausdorff space be homeomorphic to a totally complete metric space it is necessary and sufficient that it be locally compact and perfectly separable.*‡

In Theorem 1 the word metric may be replaced, on the one hand, by regular, on the other, by complete metric. This theorem is of interest chiefly because of the similar well known characterizations of the class of all compact metrisable spaces and of the class of all metrisable spaces which are homeomorphic to complete metric spaces.§ The method of proof also relates it to the two characterizations of the class of compact metrisable

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† A distinction is made between the adjectives metric and metrisable. The word metric implies the definite choice of a metric in a metrisable space. The phrase *A is topologically imbedded in B* means that *A* is homeomorphic to a subset of *B*.

‡ A totally complete metric space is a metrisable space in which the metric is so chosen that every bounded set is compact. See K. Menger, *Über die Dimension von Punktmengen*, Monatshefte für Mathematik und Physik, vol. 34 (1926), pp. 135–161; in particular, p. 144.

§ The first of these theorems is obtained by replacing, in Theorem 1, "locally compact" by "compact" and "the difference of two closed sets" by "closed." The second is obtained by replacing "locally compact" by "homeomorphic to a complete metric space" and "the difference of two closed sets" by "the intersection of a sequence of open sets." A proof of the first may be given along the lines of the proof of the first lemma, stated below. For a proof of the second, see C. Kuratowski, *Topologie I*, p. 215.