# EQUIVALENCE OF ALGEBRAIC EXTENSIONS†

BY REINHOLD BAER

The commutative fields‡ $K$ and $H$ are *equivalent* with regard to their common subfield $L$, if there exists an isomorphism between $K$ and $H$ which maps every element of $L$ upon itself. If $H$ and $K$ are equivalent with regard to $L$, then the same equations with coefficients in $L$ have solutions in $H$ and in $K$. It is the aim of this note to establish a criterion for the validity of the converse of the above proposition.

The field $F$ is *completely algebraic* with regard to its subfield $S$, if $F$ and $S$ satisfy:

(1) *$F$ is algebraic with regard to $S$;*

(2) *if $\mathfrak{f}$ is an isomorphism of $S$ upon the subfield $S'$ of the field $G'$ such that every equation (with coefficients) in $S$ which has a solution in $F$ is mapped by $\mathfrak{f}$ upon an equation in $S'$ which has a solution in $G'$, then $\mathfrak{f}$ is induced by an isomorphism of $F$ upon a field $F'$ between $S'$ and $G'$ ($S' \leqq F' \leqq G'$).*

E. Steinitz§ has proved that every simple algebraic extension|| and every normal algebraic extension¶ is completely algebraic.

LEMMA 1. *If the algebraic extension $F$ of the field $S$ satisfies the condition* (i) *to every pair of fields $U$ and $V$ such that $S \leqq U \leqq V \leqq F$, $V$ finite with regard to $U$, there exists a field $W$ between $V$ and $F$ such that $W$ is finite and completely algebraic with regard to $U$, then $F$ is completely algebraic with regard to $S$.*

PROOF. There exists a chain of fields $F_v$ ($v$ an ordinal number

---

† Presented to the Society, October 31, 1936.

‡ Only commutative fields will be considered in this note.

§ See E. Steinitz, *Algebraische Theorie der Körper*; *Mit Erläuterungen und einem Anhang*: *Abriss der Galois-schen Theorie*, neu herausgegeben von Reinhold Baer und Helmut Hasse, 1931.

|| $S$ is the simple algebraic extension of the field $F$, generated by the element $b$, if $b$ satisfies an algebraic equation with coefficients in $F$ and $S$ is a smallest field containing $F$ and $b$.

¶ $N$ is normal with regard to its subfield $S$ if every irreducible polynomial in $S$ which has zeros in $N$ is in $N$ a product of linear polynomials.