ON A CLASS OF RECURRENT SEQUENCES*

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The first example of a recurrent, non-periodic sequence was given by Marston Morse† in connection with the geodesics on a surface of negative curvature. A discussion of their significance, together with a general method of definition may be found in Birkhoff's *Dynamical Systems*, 1927, p. 246. In this note they will be considered independently of their origin, and a class of such sequences with certain interesting properties will be defined. (Theorem 1 of the present paper, except insofar as it refers to the particular sequence under discussion, is therefore not new.) As a preliminary to this we shall make certain definitions.

A *sequence* is a doubly-infinite row of the symbols 1 and 2: $\cdots c_{-3}c_{-2}c_{-1}c_0c_1c_2c_3\cdots$, where each c_i is either a 1 or a 2. A *block* is a set of consecutive members of a sequence: $c_mc_{m+1}\cdots c_n$ and has length s if there are s symbols in it. A sequence is *periodic* if there exists an integer p such that $c_{n+p}=c_n$ for all n. A sequence is *recurrent* if there exists a function of integers $f(n)$ with integral values, such that any block of length n chosen anywhere in the sequence is contained as a block in any block of length $f(n)$. The least such function $f(n)$ will be called the *ergodic function* of the sequence.

As an example of such a sequence, let

$$a_0 = 12,\quad a_1 = a_0^{-1}a_0a_0 = 21\underline{12}12,\quad \cdots,\quad a_{n+1} = a_n^{-1}a_na_n,\quad \cdots .$$

To define our sequence we number the symbols starting with the original

$$\begin{array}{c}\cdots c_0c_1\cdots\\ \cdots 12\ \cdots\end{array}$$

which has been underlined above. We may omit a more precise definition of the nth symbol since it will be unnecessary for our present purpose. We note that

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† Transactions of this Society, vol. 22 (1921), p. 94.