POLYNOMIAL APPROXIMATION ON A CURVE OF THE FOURTH DEGREE*

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1. *Introduction.* In the study of orthogonal polynomials and polynomial approximation, or of the corresponding theory for trigonometric sums, a powerful auxiliary is Bernstein's theorem on the derivative of a trigonometric sum or of a polynomial. When it is desired to investigate similar problems relating to approximation by means of polynomials on a curve in the plane of two real variables† x and y, the question arises whether something in the nature of Bernstein's theorem is available in this case also. For some of the simplest curves, such as a line segment or a circle, the question is merely one of interpretation, the polynomials in x and y reducing at once to polynomials or trigonometric sums in terms of the arc length (or a constant multiple of it) as parameter. For any curve segment of the form $x=\phi(t)$, $y=\psi(t)$, where $\phi(t)$ and $\psi(t)$ are polynomials in t, there is an immediate answer as far as differentiation with respect to t is concerned; and if $|\phi'(t)|+|\psi'(t)|$ is everywhere positive (and so has a positive lower bound) on the closed range of values of t considered, a derivative with respect to arc length does not exceed a constant multiple of the derivative with respect to t. This paper is concerned with an illustrative case in which the problem appears to be not entirely trivial, and yet susceptible of simple and elementary treatment. An appropriate form of Bernstein's theorem is obtained, and its application to a problem of polynomial approximation is indicated.

A concluding paragraph relates to the convergence in the mean of developments in series of orthogonal polynomials on an arbitrary curve.

2. *Bernstein's Theorem.* Let C be the curve

$$x^4 + y^4 = 1.$$

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† See D. Jackson, *Orthogonal polynomials on a plane curve*, presently to appear in the Duke Mathematical Journal.