$$F_{89}(x_8) = f(x_8) \leqq F_{k_0}(x_8),$$
$$F_{89}(x_9) = f(x_9) < F_{k_0}(x_9),$$

so that, by Theorem 1 and its corollary,

$$F_{89}(x) < F_{k_0}(x), \qquad (x_8 < x < b);$$

in particular,

$$F_{89}(x_7) < F_{k_0}(x_7). \tag{41}$$

Now (41) contradicts (39) and (40).

THE RICE INSTITUTE

SUFFICIENT CONDITIONS FOR A NON-REGULAR PROBLEM IN THE CALCULUS OF VARIATIONS*

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1. *Introduction*. Given $J=\int_{x_1}^{x_2} f(x, y, y')dx$, it is well known that a minimizing curve satisfies the necessary conditions of Euler, Weierstrass, and Legendre, which we shall designate as I, II, and III,† respectively. If further, $f_{y'y'}(x, y, y') \neq 0$ on the minimizing curve, the Jacobi condition IV is necessary, while the stronger set of conditions I, II_b', III′, and IV′‡ are sufficient for a strong relative minimum.

The purpose of this study is to obtain a set of sufficient conditions for a curve without corners along which $f_{y'y'}$ may have zeros. Since the classical theory gives only the necessary conditions I, II, and III, we wish to obtain a Jacobi condition; and with this in view, introduce the integral

$$L \equiv \int_{x_1}^{x_2} \phi(x, y, y')dx, \ \phi(x, y, y') \equiv f(x, y, y') + k^2[y' - e'(x)]^2, \qquad (x_1 \leqq x \leqq x_2, k \lesseqgtr 0),$$

by means of which we find a necessary condition that we shall call IV_L'. Suitably strengthened, this becomes IV_{Lb}' and the set of conditions I, II_b, III_b, and IV_{Lb}' are found sufficient for an improper strong relative minimum.

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† G. A. Bliss, *Calculus of Variations*, 1925, pp. 130–132.

‡ Bliss, loc. cit., pp. 134–135.