THE ROLE OF ALGEBRA IN TOPOLOGY†

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1. *Introduction.* The assertion is often made of late that all mathematics is composed of algebra and topology. It is not so widely realized that the two subjects interpenetrate so that we have an algebraic topology as well as a topological algebra.

The increasing importance of algebra in topology, a domain whose roots lie in a very different soil, signifies that here also the age of consolidation and coordination is upon us. My present object is primarily to show that a reasonable blend of the algebraic and topological points of view is possible. For this purpose I shall formalize my earlier procedure of immersion in variable euclidean spaces by means of which I introduced dual cycles into topology. We shall see that around this mode of attack it is possible to group many of the recent very interesting results of combinatorial topology.

I. Algebra of Complexes

2. *Abstract Complexes.* Abstract complexes have been investigated by various authors, notably in recent years by J. W. Alexander, W. Mayer, and A. W. Tucker.‡ While I shall lean particularly on Tucker's work, my discussion bears largely on simplicial complexes and their duals, the basic types in topology.

According to Tucker, then, an *abstract* complex K is a set of elements E, its cells, partially ordered relative to a transitive geometric relation of incidence $<$ ("on the boundary of"), and with certain additional (algebraic) relations of incidence to be described presently. Each E has a dimension p which is a positive or negative integer and shall be frequently denoted by an index, as E_p. Moreover $E < E'$ implies $\dim E < \dim E'$. A *p-chain* of K is a linear form

$$C_p = x_i E_p^i, \tag{1}$$

† An address delivered at Duke University, December 30, 1936, as the retiring presidential address, before the American Mathematical Society.

‡ J. W. Alexander, Transactions of this Society, vol. 28 (1926), pp. 301–329; W. Mayer, Monatshefte für Mathematik und Physik, vol. 36 (1929), pp. 1–42, 219–258; A. W. Tucker, Annals of Mathematics, vol. 34 (1933), pp. 191–243.