SOME FORMULAS FOR FACTORABLE POLYNOMIALS IN SEVERAL INDETERMINATES†

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1. *Introduction.* By a factorable polynomial‡ in the $GF(p^n)$ will be meant a polynomial in the indeterminates $x_1, \cdots, x_k$, which factors into a product of linear factors in some (sufficiently large) Galois field:

$$G \equiv G(x_1, \cdots, x_k) \equiv \prod_{j=1}^{m} (\alpha_{j0} + \alpha_{j1}x_1 + \cdots + \alpha_{jk}x_k).$$

It is frequently convenient to consider separately those G (of degree m) in which x_k^m (or any assigned x_i^m) actually occurs; we use the notation G^* to denote such a polynomial. In the case $k=1$, the polynomials G reduce to ordinary polynomials in a single indeterminate; in this case G and G^* are identical.

In this note we extend certain results§ for $k=1$ to the case $k>1$. For polynomials G^* the extensions may (roughly) be obtained by merely replacing p^n by p^{nk}; for arbitrary G the generalizations are not quite so simple.

2. *The μ-Function.* For G of degree m, we put $|G| = p^{nm}$; then

$$\zeta^*(w) = \sum_{G^*} \frac{1}{|G|^w} = (1 - p^{n(k-w)})^{-1}, \tag{1}$$

$$\begin{aligned} \zeta(w) &= \sum_{G} \frac{1}{|G|^w} \\ &= \left\{(1 - p^{n(1-w)})(1 - p^{n(2-w)}) \cdots (1 - p^{n(k-w)})\right\}^{-1}, \end{aligned} \tag{2}$$

the sums extending over *all* G^*, G, respectively.

Let $f(m)$ be the number of (non-associated) G of degree m, $f^*(m)$ the number of G^*; from the first of these formulas it follows that $f^*(m) = p^{nkm}$, and from the second, $f(m) = [k+m-1, m]p^{nm}$, where

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‡ Duke Mathematical Journal, vol. 2 (1936), pp. 660–670.

§ American Journal of Mathematics, vol. 54 (1932), pp. 39–50; this Bulletin, vol. 38 (1932), pp. 736–744.