# AN APPLICATION OF DERIVATIVES OF NON-ANALYTIC FUNCTIONS IN PLANE STRESS PROBLEMS*

BY V. P. JENSEN AND D. L. HOLL

1. *The Plane Stress Problem.* A plane state of stress is defined in a region in the $xy$ plane by an Airy's stress function $F(x, y)$, where $F$ satisfies

$$\nabla^4 F(x, y) = \nabla^2\nabla^2 F = \nabla^2(F_{xx} + F_{yy}) = 0. \tag{1}$$

The normal stresses $\sigma_x$ and $\sigma_y$ in the directions of $x$ and $y$, respectively, and the corresponding shearing stress $\tau_{xy}$ are obtained from $F$ when no body forces are present by the following:†

$$\sigma_x = F_{yy}, \qquad \sigma_y = F_{xx}, \qquad \tau_{xy} = -F_{xy}. \tag{2}$$

Equilibrium conditions show that the stress tensor at any point may be referred to any set of orthogonal planes by the relations

$$\begin{aligned} \sigma_{x'}, \sigma_{y'} &= \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sigma_x - \sigma_y}{2} \cos 2\theta \pm \tau_{xy} \sin 2\theta, \\ \tau_{x'y'} &= \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \cos 2\theta, \end{aligned} \tag{3}$$

where the $x'y'$ axes have been rotated through the positive angle $\theta$ from the $xy$ axes.

2. *The Stress Circle and Kasner's Derivative Circle.* A graphical construction due to Mohr‡ is frequently employed in place of equations (3). In the complex plane $\gamma = \sigma + i\tau$, describe a circle having its center on the $\sigma$ axis and passing through the points $(\sigma_x, \tau_{xy})$ and $(\sigma_y, -\tau_{xy})$ which are designated respectively as points $C$ and $E$. Then, corresponding to a counter-clockwise ro-

* Presented to the Society, November 28, 1936.

† Subscripts on $F$ denote partial derivatives with respect to the indicated variables. The subscripts on the stresses $\sigma$ and $\tau$ refer to directions along which the stresses act.

‡ O. Mohr, *Abhandlungen aus dem Gebiete der Technische Mechanik*, 2d edition, 1914.