ON ARITHMETIC INVARIANTS OF BINARY CUBIC AND BINARY TRILINEAR FORMS*

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1. *Introduction.* It is the purpose of this note to show that binary cubic and binary trilinear forms can be completely and very simply characterized by *arithmetic rank invariants* for non-singular linear transformations in the complex field. We define the factorization rank† of a matrix $A = (a_{ijk})$ of order n and its associated trilinear form to be the minimum value of ϵ such that A can be "factored" into the form

$$(1) \qquad A = \left(\sum_{\alpha=1}^{\epsilon} a_{\alpha i} b_{\alpha j} c_{\alpha k} \right), \qquad (i, j, k = 1, 2, \cdots, n).$$

Hitchcock‡ obtained minimum values of ϵ for certain polyadics. The number ϵ is invariant under non-singular linear transformations on the variables in the trilinear form

$$\sum a_{ijk} x_i y_j z_k$$

associated with A. In a paper which appeared in this Bulletin,§ I classified binary trilinear forms by means of ranks and a property of invariant factors, which are invariant under non-singular linear transformations in the complex field. *The ranks of that paper and the rank defined above form a complete invariant system for these forms. For binary cubic forms the factorization rank alone forms a complete invariant system.*

2. *Factorization Ranks.* The canonical binary trilinear forms are

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† This rank was used in developing a general theory of non-singular p-way matrices in the paper *Non-singular multilinear forms and certain p-way matrix factorizations*, Transactions of this Society, vol. 39 (1936), pp. 422–455. If factorization rank is similarly defined for 2-way matrices, it is found that the factorization rank of a 2-way matrix is n if and only if its ordinary rank is n.

‡ F. L. Hitchcock, *The expression of a tensor or polyadic as a sum of products*, Journal of Mathematics and Physics, Massachusetts Institute of Technology, vol. 6 (1927), pp. 164–189.

§ *On canonical binary trilinear forms*, vol. 38 (1932), pp. 385–387.