NOTE ON FORMULAS FOR THE NUMBER OF REPRESENTATIONS OF AN INTEGER AS A SUM OF $2h$ SQUARES*

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1. *Introduction.* It is known that the number of representations of an arbitrary integer n as a sum of $2h$ squares may be expressed in the form

$$\lambda \sum (-1)^{(hd-h)/2} d^{h-1}, \tag{1}$$

when $1 \leqq h \leqq 4$. The number λ may depend on the linear form of n and the summation is over all *odd* divisors d of n. For example, an odd integer can be represented as a sum of four squares in $8\sum d$ ways, while an even integer can be represented in $24\sum d$ ways. The question arises whether formulas of the type (1) hold when $h \geqq 5$. This question has been answered in the negative for $n = m$ or $n = 2m$, where m is odd, by E. T. Bell† and in some cases for $n = 2^\alpha m$, $\alpha \geqq 1$ by the author.‡ These results were obtained by the use of the theory of elliptic functions.

In this note we shall prove a necessary condition which does not require any elliptic function theory and which enables us to decide if a formula of the type (1) is possible or not. It is to be noticed that this method does not show when formulas of the type (1) are true but only when they are *not* true.

2. *Necessary Condition for Formulas of Type* (1). Let n_j denote any one of a given class $K(n_j)$ of positive integers in which $n_j < n_{j+1}$. For example, we might take $n_j = 4j - 3$ and then $K(n_j)$ would be the class of all positive integers which are $\equiv 1 \pmod 4$. Let $N(n, 2h)$ denote the number of representations of an integer n as a sum of $2h$ squares, both the arrangement of the squares and the signs of their square roots being relevant in counting the representations. Then we have the following result, which

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† Journal für die reine und angewandte Mathematik, vol. 163 (1930), pp. 65–70; Journal of the London Mathematical Society, vol. 4 (1929), pp. 279–285.

‡ American Journal of Mathematics, vol. 58 (1936), pp. 536–544.