A NOTE ON A PRECEDING PAPER*

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1. *Introduction.* In a paper† by the author, the following lemma was proved.

LEMMA. *If X_s is an element of $A(p)$, the number $\{\sum_{q=1}^{n}(q/n)X_s\vee\}$ is a member of the set $A[1-(1-p)^n]$.*

Then again,‡ the author applied a theorem due to Copeland§. It is our purpose here to extend these two theorems to apply in the field of geometrical probability. The proof of the theorem corresponding to Copeland's follows a different procedure from that given by him. As a matter of fact, the theorem of Copeland may be proved by the method given here.

2. *Extension of the Lemma.* The extension is as follows.

THEOREM 1. *If the numbers $(q/n)x(E_q)$, $(q=1, 2, \cdots, n)$, are such that $x(E_q) = \cdot\phi_{E_q}(P_1), \phi_{E_q}(P_2), \cdots$, where E_q is the interval $0<y\leqq p_q$ and $P_1, P_2, \cdots$ is a set of points admissibly ordered with respect to the function $m(E)$ (the Lebesgue measure of E) defined in Δ; $0<y\leqq 1$, then (1) the number $\sum_{q=1}^{M}(q/n)x(E_q)\vee$ has the probability $[1-\prod_{q=1}^{M}(1-p_q)]$ and (2) the number $\sum_{q=1}^{M}(q/n)x(E_q)\vee$ is a member of the set $A[1-\prod_{q=1}^{M}(1-p_q)]$, where $M\leqq n$.*

PROOF OF (1). We know that||

$$\text{(a)} \qquad \sim\sum_{q=1}^{M}\left(\frac{q}{n}\right)x(E_q)\vee = \prod_{q=1}^{M}\sim\left(\frac{q}{n}\right)x(E_q)\cdot .$$

* Presented to the Society, February 29, 1936.

† See the author's memoir *The application of the theory of admissible numbers to time series with constant probability*, Transactions of this Society, vol. 36 (1934), p. 517.

‡ Same reference as above, p. 524.

§ See Copeland, *Admissible numbers in the theory of probability*, American Journal of Mathematics, vol. 50 (1928), p. 550, Theorem 16.

|| The symbol $\sum(q/n)x(E_q)\vee$ represents the number $\{(1/n)x(E_1)\vee\cdots\vee(M/n)x(E_M)\vee\}$, while $\Pi\sim(q/n)x(E_q)\cdot$ represents $\{\sim(1/n)x(E_1)\cdot\cdots\cdot(M/n)x(E_M)\cdot\}$. Throughout the paper such symbols will have similar mean-