## **INSTRUMENTAL ANALYSIS\***

## BY VANNEVAR BUSH

The use of instruments of computation and analysis is as old as mathematics itself. Counting by the aid of piles of pebbles gave origin to the word calculus. There have always been such aids to reasoning, and the development of instrumental methods has throughout proceeded in parallel with formal methods.

Under instrumental analysis is to be grouped all analysis proceeding by the use of devices for supplementing pure reasoning, whether these devices involve mechanics, optics, heat, electricity or other natural phenomena. The device aids the mind because it approximately obeys some simple law, and may be made to indicate the consequences of combinations of such relationships. An example is the ordinary compass which will draw an approximate circle. One may reason about the properties of families of circles and never draw a diagram; but such reasoning is more surely guided by a few pictures. The instrument, whatever it may be, has two functions: first, when approximate results are sufficient, to yield these directly; second, as a suggestive auxiliary to precise reasoning.

The first piece of mathematical apparatus used was the human hand. Nature, for some reason that is not entirely clear, supplied us with ten fingers, and in the groping development that characterized the beginnings of mathematics, this settled the decimal system upon us. Our forebears apparently did their reckoning standing up, or we might, as in fact did more than one tribe, have used toes as well, and thus landed on the vicenary system. Had this occurred it would have been possible to register all of New York's automobiles with five figures; and Congress would probably have had to be satisfied with merely eight figure appropriations. It is certainly fortunate that at least both hands were used, or the newspapers would now be running out of zeros.

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