to whether the book should be destined for the beginner or whether its purpose should rather be a survey of a field for experts and fellow-contributors. A little contrast can be doubtless observed between the broad treatment of some comparatively elementary situations and the preference of special and individual topics to some others of importance. But at all events the book will be useful from both points of view and it is a significant enrichment of the series of the Colloquium Lectures.

GABRIEL SZEGÖ

MORSE ON CALCULUS OF VARIATIONS

The Calculus of Variations in the Large. By Marston Morse. Colloquium Publications of the American Mathematical Society, vol. 18. New York, 1934. ix+368 pp.

The background for the theory elaborated in this volume lies in two rather distinct fields of mathematics. We have on the one hand the theory of critical points of functions of n real variables, largely created and developed by the author and his students; on the other hand, the classical calculus of variations and its modern treatment as a part of the functional calculus, to which Hadamard and Tonelli have made the fundamental contributions. The origin of the theory of critical points is perhaps to be found in the minimax principle introduced by Birkhoff in his paper on *Dynamical systems with two degrees of freedom* (see Transactions of this Society, vol. 18 (1917), p. 240). While at this early stage the connection of the theory with the calculus of variations was made clear, later developments have made the relations between the two fields much more intimate.

The calculus of variations in the large derives its interest not only from its use of functional calculus methods. More significant is the fact that it undertakes a study of the configurations to which a calculus of variations problem gives rise, namely, the extremals, with respect to important properties apart from the question whether or not they furnish extreme values for a definite integral. In the classical calculus of variations, only such extremal arcs AB were considered which contained no point A' conjugate to A. The theory with which this book is concerned, and also the earlier investigations of Birkhoff, reveal the importance of extremal arcs AB upon which there may be one or more points conjugate to A; not only arcs in the small, restricted by the exclusion of conjugate points, but also arcs in the large have to be considered for a full understanding of this theory.

A critical point x_i^0 of $f(x_1, \cdots, x_m)$ is a point at which all the first order partial derivatives of f vanish. Each critical point has a type number equal to the number of negative terms in the quadratic form which represents the second order terms of $f(x_i)-f(x_i^0)$, when reduced to a sum of squares. The author's earlier papers establish a beautifully simple set of relations between the numbers of critical points of various types of $f(x_1, \cdots, x_m)$ and the connectivity numbers of the domain R over which the function ranges. Thus the topological character of the domain of the independent variable is linked up