SOME PROPERTIES OF THE DISCRIMINANT MATRICES OF A LINEAR ASSOCIATIVE ALGEBRA*

BY R. F. RINEHART

1. *Introduction.* Let A be a linear associative algebra over an algebraic field. Let $e_1, e_2, \cdots, e_n$ be a basis for A and let c_{ijk}, $(i, j, k=1, 2, \cdots, n)$, be the constants of multiplication corresponding to this basis. The first and second discriminant matrices of A, relative to this basis, are defined by

$$T_1(A) = \|t_1(e_re_s)\| = \left\| \sum_{i,j=1}^{n} c_{rsi}c_{ijj} \right\| = \left\| \sum_{i,j=1}^{n} c_{rij}c_{sji} \right\|,$$

$$T_2(A) = \|t_2(e_re_s)\| = \left\| \sum_{i,j=1}^{n} c_{rsi}c_{jij} \right\| = \left\| \sum_{i,j=1}^{n} c_{irj}c_{jsi} \right\|,$$

where $t_1(e_re_s)$ and $t_2(e_re_s)$ are the first and second traces, respectively, of e_re_s. The first forms in terms of the constants of multiplication arise from the isomorphism between the first and second matrices of the elements of A and the elements themselves. The second forms result from direct calculation of the traces of $R(e_r)R(e_s)$ and $S(e_r)S(e_s)$, $R(e_i)$ and $S(e_i)$ denoting, respectively, the first and second matrices of e_i. The last forms of the discriminant matrices show that each is symmetric.

E. Noether† and C. C. MacDuffee‡ discovered some of the interesting properties of these matrices, and shed new light on the particular case of the discriminant matrix of an algebraic equation. It is the purpose of this paper to develop additional properties of these matrices, and to interpret them in some familiar instances.

Let A be subjected to a transformation of basis, of matrix M,

$$e_i = \sum_{j=1}^{n} m_{ij}e_j, \qquad (i = 1, 2, \cdots, n; m = |m_{rs}| \neq 0).$$

* Presented to the Society, November 30, 1935. This paper, with proofs and details not included here, is on file as a doctoral thesis in the Library of the Ohio State University.

† Mathematische Zeitschrift, vol. 30 (1929), p. 689.

‡ Annals of Mathematics, (2), vol. 32 (1931), pp. 60–66; and Transactions of this Society, vol. 33 (1931), pp. 425–432.