INVOLUTORIAL SPACE TRANSFORMATIONS ASSOCIATED WITH A RATIONAL RULED SURFACE*

BY L. A. DYE

1. *Introduction.* The present paper concerns various new types of Cremona involutorial transformations in S_3. Each one is a sample of an infinite category, and the concept can be extended to higher spaces.

Transformations obtained by joining corresponding points of curves or by associating the points of a curve with a projective pencil of surfaces have been studied from time to time and are sketched in the *Encyklopädie*, the *Repertorium*, and particularly in *Selected Topics of Algebraic Geometry*, including the Supplementary Report (Bulletin 63, National Research Council, 1928, No. 96, 1934). All these papers can be found in *Topics*, Chapters 8, 9, or Supplement, Chapters 4, 5 under the names Black, Carroll, Davis, DePaoli, Dye, Moffa, Montesano, Sharpe, and Snyder.

The types here discussed can not be put into any of the categories previously mentioned. The complex of lines defined by pairs PP' of associated points is not linear, is not special, and does not have any particular role in the problem. In most of the earlier cases it was formed by the secants to a given curve, or was linear. In the DePaoli types the lines PP' describe a congruence, each line containing an infinite number of pairs of conjugate points.

The procedure used in this paper is to establish a (1, 1) correspondence between the generators of a ruled surface R and the surfaces of a pencil $|F|$. A general point P of space selects a surface F of the pencil and hence the associated generator r of R. The plane π determined by r and P is tangent to R at a point Q. The line PQ meets the surface F in P and a residual point P'; any other intersections are accounted for by properly relating $|F|$ and R. The points P, P' are an associated pair in an involutorial transformation under which the pencil $|F|$ and the congruence of conics cut from $|F|$ by the planes π are invariant.

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