ON CERTAIN DISTRIBUTION THEOREMS
OF STATISTICS*

BY SOLOMON KULLBACK

1. *Introduction.* The problems of statistics have been classified under three heads: problems of specification, problems of estimation, problems of distribution.† I intend to prove certain theorems regarding the problem of distribution in more general form than that in which they have been proved before.

By the distribution problem of statistics is meant the problem of finding the distribution function, or law of total probability, of the real, single-valued functions $\mu_\alpha(x_1, x_2, \cdots, x_n)$, $(\alpha = 1, 2, \cdots, r)$, given the distribution function of the real variables $x_1, x_2, \cdots, x_n$. Many particular problems of this nature have been solved. More general solutions have been obtained, imposing certain restrictions on the functions $\mu_\alpha(x_1, x_2, \cdots, x_n)$ and the distribution function of $x_1, x_2, \cdots, x_n$.‡

2. *Distribution Functions.* By the distribution function, or law of total probability of the real variables $x_1, x_2, \cdots, x_n$, is meant the monotone, absolutely additive set function $\phi(E)$, where $0 \leq \phi(E) \leq 1$, and $\phi(S) = \int_S d_{x_1 x_2} \ldots_{x_n} \phi(E) = 1$, where S is the whole $x_1 x_2 \cdots x_n$ space.§

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† R. A. Fisher, *On the mathematical foundations of theoretical statistics,* Philosophical Transactions of the Royal Society, vol. 222 A (1922), p. 309 ff.

‡ C. V. L. Charlier, *Some fundamental problems in the theory of probability,* Arkiv för Matematik, Astronomi och Fysik, vol. 8 (1912), No. 4, pp. 1–39; T. Kameda, I. *Eine Verallgemeinerung des Poissonschen Problems in der Wahrscheinlichkeitsrechnung,* Proceedings Physico-Mathematical Society, Tokyo, vol. 9 (1917–18), pp. 155 ff.; II. *Theory of generating functions and its application to the theory of probability,* Journal Faculty of Science, University of Tokyo, sec. I, vol. 1 (1925), part 1, pp. 1–62; E. L. Dodd, *The frequency law of a function of variables with given frequency laws,* Annals of Mathematics, (2), vol. 27 (1925), pp. 12–20; S. Kullback, *An application of characteristic functions to the distribution problem of statistics,* Annals of Mathematical Statistics, vol. 5 (1934), pp. 263–307.

§ See E. K. Haviland, I. *On the theory of absolutely additive distribution functions,* American Journal of Mathematics, vol. 56 (1934), pp. 625–658, particularly pp. 627–628.