SIERPINSKI ON THE CONTINUUM

Hypothèse du Continu. By W. Sierpinski. (Monografje Matematyczne, Tome 4.) Warsaw, Seminarjum Matematyczne Uniwersytetu Warszawskiego, 1934. v+192 pp.

This book is the fourth of the excellent group of monographs which are appearing under the auspices of the Mathematical Seminar of the University of Warsaw. In it Professor Sierpinski makes an exhaustive study of Cantor's hypothesis of the continuum, which may be stated in an elementary form as the hypothesis that every non-denumerable set of real numbers has the power of the continuum. The form of the hypothesis of the continuum the author most generally uses is that of the equality $2^{\aleph_0}=\aleph_1$. He points out that the known demonstrations of the equivalence of these two forms of the hypothesis require the use of the axiom of choice. He distinguishes between the hypothesis of the continuum and the problem of the continuum which is that of finding the place of the continuum amongst the alephs, that is, the problem of determining the ordinal α such that $2^{\aleph_0}=\aleph_\alpha$. If the hypothesis of the continuum were true one would have $\alpha=1$. That α is not ω is a result due to J. König; Sierpinski points out that König's demonstration may be modified so as not to use the axiom of choice.

The author calls attention to the fact that important simplifications would be made in point set theory were one able to demonstrate the hypothesis of the continuum. Many important propositions of different branches of mathematics cannot be deduced at present without using this hypothesis. For persons not recognizing the validity of reasoning based on this hypothesis, it is of extreme importance to know the propositions in whose proof it is used and which we actually do not know how to prove without it.

In Chapter 1 there are given eleven propositions each of which is equivalent to the Hypothesis H. Among those which are the most interesting to the reviewer and also most frequently used in the remainder are the following: P_1, which states that the plane is the sum of two sets, one of which is at most denumerable on every parallel to the axis of ordinates, and the other at most denumerable on every parallel to the axis of abscissas; P_3, to the effect that there exists an infinite sequence of single-valued functions of $x, f_1(x), f_2(x), \cdots$ such that given any non-denumerable set N of real numbers, all functions of the sequence, save possibly a finite number, transform N into the set of all real numbers; P_7, which states that there exists an analytic linear set which is not the sum of less than $2^{\aleph_0}$ sets measurable (B).

In Chapter 2 the author is concerned mainly with the sets of Lusin, that is, sets of the power of the continuum which have at most a countable set in common with each linear non-dense perfect set. Lusin demonstrated with the aid of the hypothesis of the continuum that these sets, which we shall call sets L, exist. Among the interesting properties of these sets that are pointed out are: (1) Every non-denumerable subset is of second category of Baire; (2) Every function of Baire defined on it is of class $\leqq 2$ on the set; (3) Every function of Baire transforms a set with property L into one with property C,