ON THE PRINCIPLES OF HAMILTON AND CARTAN

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1. *Introduction.* A holonomic dynamical system can be characterized by a Hamilton stationary action integral or by a Cartan integral invariant. A. E. Taylor* has extended Cartan's principle to the case of non-holonomic systems. It is the purpose of this paper to obtain by a different method both Taylor's extension of Cartan's principle and the corresponding extension of Hamilton's principle. The latter is an extension in a sense not hitherto obtained.

2. *Extensions to Non-holonomic Systems.* As we shall make the extensions by transforming non-holonomic systems into equivalent holonomic systems, we shall first state the principles for holonomic systems in the form which is most suitable for our purposes.

Suppose we have a dynamical system defined by the equations

$$\dot{q}_r = \frac{\partial H}{\partial p_r}, \qquad \dot{p}_r = -\frac{\partial H}{\partial q_r}, \qquad (r = 1, \cdots, k). \tag{1}$$

Then in the qpt space these equations determine a $(2k)$-parameter family of trajectories.

To make use of the notation of Cartan,† we shall denote the parameters of the system by $\alpha, \beta_1, \cdots, \beta_{2k-1}$, where the parameters are such that for the β's constant the trajectories given by $\alpha_0 \leqq \alpha \leqq \alpha_1$ form a tube of trajectories.

Also a parameter u is introduced such that $dt = \rho du$, where ρ is a function of u, α, and the β's, arbitrary except that it is always of the same sign.

When the β's and u are fixed and α varies from α_0 to α_1, then a locus of corresponding points on the tube determined by the β's is obtained. The locus is arbitrary owing to the presence of ρ,

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* A. E. Taylor, *On integral invariants of non-holonomic dynamical systems*, this Bulletin, vol. 40 (1934), pp. 735–742.

† E. Cartan, *Leçons sur les Invariants Intégraux*, 1922.